(6)
MOSFET Differential Pair

Reference: Neamen, Chapter 11

Learning Outcome

Able to:

- Describe the mechanism by which a differential-mode signal and common-mode signal are produced in a MOSFET differential-amplifier.
- Describe the dc transfer characteristics of a MOSFET differential-amplifier.

6.0) Essential Formulas

Region	NMOS	PMOS
Saturation	$v_{DS} \geq v_{DS}$(sat) $i_D = K_n[v_{GS} - V_{TN}]^2$	$v_{SD} \geq v_{SD}$(sat) $i_D = K_p[v_{SG} + V_{TP}]^2$
Transition Point	v_{DS}(sat) = v_{GS} - V_{TN}	v_{SD}(sat) = v_{SG} + V_{TP}

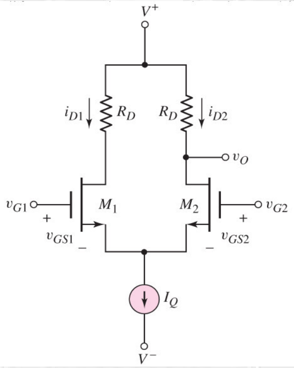
$$K_n = \frac{\mu_n C_{ox} W}{2L} = \frac{k_n'}{2} \cdot \frac{W}{L}$$

$$K_p = \frac{\mu_p C_{ox} W}{2L} = \frac{k_p'}{2} \cdot \frac{W}{L}$$

$$C_{ox} = \varepsilon_{ox} / t_{ox}$$

μ_n, μ_p	Mobility of electrons, **holes**
ε_{ox}	Oxide permittivity
t_{ox}	Oxide thickness
W, L	Channel **W**idth, **L**ength
$k_n' = \mu_n C_{ox}$ $k_p' = \mu_p C_{ox}$	Process conduction parameter (provided by manufacturer)

6.1) Basic MOSFET Differential Pair



- **Figure 11.19** shows the basic MOSFET diff-pair configuration using N-MOSFET.

Figure 11.19: Basic MOSFET differential-pair configuration.

6.1) Basic MOSFET Differential Pair (Cont)

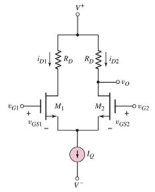


Figure 11.19: Basic MOSFET differential-pair configuration.

- **Two identical transistor M_1 and M_2**, whose sources are connected together, are biased by a constant current source I_Q which is connected to a negative supply V^-
- The drains of M_1 and M_2 are connected through R_D to a positive supply V^+
- Assume that **M_1 and M_2 are always biased in the saturation region**. Even with $v_{G1} = v_{G2} = 0$, the M_1 and M_2 can be biased in saturation region **by the current source I_Q**.

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8

Objective:
Calculate the dc characteristics of a MOSFET diff-amp.

Consider the diff-amp shown in **Figure 11.20**. The transistor parameters are:

$K_{n1} = K_{n2} =$ **0.1 mA/V^2**,
$K_{n3} = K_{n4} =$ **0.3 mA/V^2**,
and for all transistors
$\lambda =$ **0** and $V_{TN} =$ **1 V**

Determine the maximum range of common-mode input voltage, i.e. find v_{CM} **(max)** and v_{CM} **(min)**.

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)

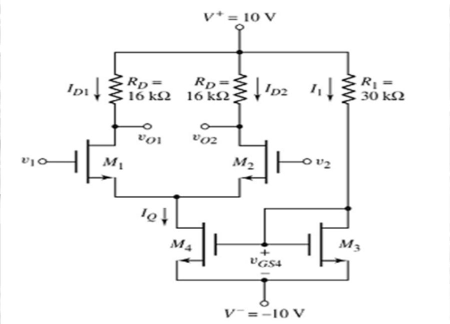


Figure 11.20: MOSFET diff amp for Example 11.8

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)

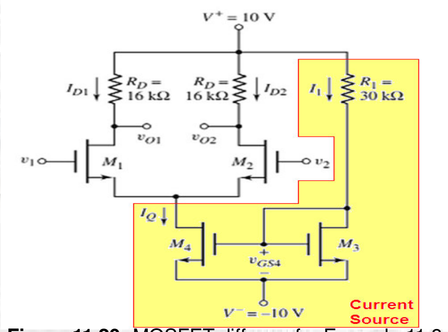


Figure 11.20: MOSFET diff amp for Example 11.8

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)

Solution:
The **reference current** can be determined from:

$$I_1 = \frac{V^+ - V^- - V_{GS4}}{R_1} = \frac{20 - V_{GS4}}{R_1}$$

and from $I_1 = K_{n3}(V_{GS4} - V_{TN})^2$

Thus $9V_{GS4}^2 - 17V_{GS4} - 11 = 0$ $\quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Yields $V_{GS4} = 2.40\text{V}$

and $I_1 = 0.587\text{mA}$

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)

The **quiescent drain currents** in M_1 and M_2:

$I_{D1} = I_{D2} = I_Q / 2 \approx$ **0.293 mA**

The gate-to-source voltages are then

$$V_{GS1} = V_{GS2} = \sqrt{\frac{I_{D1}}{K_{n1}}} + V_{TN} = \sqrt{\frac{0.293}{0.1}} + 1 = 2.71\text{V}$$

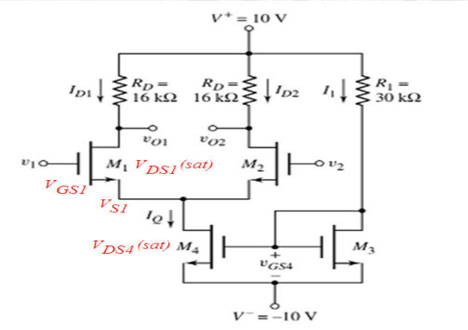
The quiescent values of v_{O1} and v_{O2} are

$v_{O1} = v_{O2}$ $= V^+ - I_{D1} R_D / 2$

$= 10 - (0.293\text{m})(16\text{k})/2 =$ **5.31 V**

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)



6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)

The maximum common-mode input voltage is the value when M_1 and M_2 reach the transition point

v_{CM}**(max)** $= V_{S1}$**(max)** $+ V_{GS1}$
$= (v_{O1} - V_{DS1}(\text{sat})) + V_{GS1}$

The minimum common-mode input voltage is the value when M_4 reaches the transition point

v_{CM}**(min)** $= V_{S1}$**(min)** $+ V_{GS1}$
$= (V^- + V_{DS4}(\text{sat})) + V_{GS1}$

6.1) Basic MOSFET Differential Pair (Cont)

Example 11.8 (Cont)

The maximum common-mode input voltage is the value when M_1 **and** M_2 **reach the transition point**,

$V_{DS1} = V_{DS2} = V_{DS1}(\text{sat}) = V_{GS1} - V_{TN}$

$V_{DS1} = 2.71 - 1 = \mathbf{1.71\ V}$

Therefore,

$v_{CM}\,(\text{max}) = v_{O1} - V_{DS1}(\text{sat}) + V_{GS1} = 5.31 - 1.71 + 2.71 = \mathbf{6.31\ V}$

The minimum common-mode input voltage is the value when M_4 **reaches the transition point**,

$V_{DS4} = V_{DS4}(\text{sat}) = V_{GS4} - V_{TN} = 2.4 - 1 = \mathbf{1.4\ V}$

Therefore,

$v_{CM}\,(\text{min}) = V^- + V_{DS4}(\text{sat}) + V_{GS1} = (-10) + 1.4 + 2.71 = \mathbf{-5.89\ V}$

6.2) DC Transfer characteristics

- Use circuit in **Figure 11.19**. Assume M_1 and M_2 are matched and neglecting their output resistances:

$$i_{D1} = K_n\left(v_{GS1} - V_{TN}\right)^2 \qquad (11.60(a))$$

$$i_{D2} = K_n\left(v_{GS2} - V_{TN}\right)^2 \qquad (11.60(b))$$

Taking square roots and subtracting

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{K_n}\left(v_{GS1} - v_{GS2}\right) = \sqrt{K_n}.v_d$$

where $v_d = v_{G1} - v_{G2} = v_{GS1} - v_{GS2}$ (11.61)

is the **differential-mode input voltage.**

6.2) DC Transfer characteristics (Cont)

- If $v_d > 0$, then $v_{G1} > v_{G2}$ and $v_{GS1} > v_{GS2}$ which implies that $i_{D1} > i_{D2}$
- Since $i_{D1} + i_{D2} = I_Q$ (11.62)

$$\left(\sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}}\right)^2 = \left(\sqrt{K_n}.v_d\right)^2 = K_n v_d^2$$

$$\sqrt{i_{D1}\left(I_Q - i_{D1}\right)} = \frac{1}{2}\left(I_Q - K_n v_d^2\right)$$

$$i_{D1}^2 - I_Q i_{D1} + \frac{1}{4}\left(I_Q - K_n v_d^2\right)^2 = 0$$

6.2) DC Transfer characteristics (Cont)

- Applying the **quadratic formula**, rearranging terms, and noting that $i_{D1} > I_Q/2$ and $v_d > 0$, can obtain

$$i_{D1} = \frac{I_Q}{2} + \sqrt{\frac{K_n I_Q}{2}}.v_d\sqrt{1 - \left(\frac{K_n}{2I_Q}\right)v_d^2} \qquad (11.66)$$

- Using Equation (11.62) can also obtain

$$i_{D2} = \frac{I_Q}{2} - \sqrt{\frac{K_n I_Q}{2}}.v_d\sqrt{1 - \left(\frac{K_n}{2I_Q}\right)v_d^2} \qquad (11.67)$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6.2) DC Transfer characteristics (Cont)

- The normalized drain currents are

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q}}.v_d\sqrt{1 - \left(\frac{K_n}{2I_Q}\right)v_d^2} \qquad (11.68)$$

and

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \sqrt{\frac{K_n}{2I_Q}}.v_d\sqrt{1 - \left(\frac{K_n}{2I_Q}\right)v_d^2} \qquad (11.69)$$

➔ These equations describe the dc transfer characteristics for the circuit. They are plotted in **Figure 11.21** as a function of normalized differential input voltage $v_d / \sqrt{\left(2I_Q / K_n\right)}$

6.2) DC Transfer characteristics (Cont)

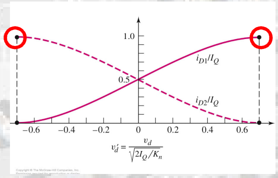


Fig 11.21: Normalized dc transfer characteristics for MOSFET differential amp.

- From (11.68) and (11.69), at a specific differential input voltage, bias current I_Q is switched entirely to one transistor or the other.

➔ **This occurs when** $\left|v_d\right|_{\max} = \sqrt{I_Q / K_n}$ (11.70)

6.2) DC Transfer characteristics (Cont)

- Forward transconductance = the slope of the dc transfer characteristics for the i_{D1} curve.
- From **Figure 11.21**, maximum forward transconductance occurs at $v_d = 0$, so that

$$g_f(\max) = \left.\frac{di_{D1}}{dv_d}\right|_{v_d=0} \qquad (11.71)$$

- Using (11.66),

$$g_f(\max) = \sqrt{\frac{K_n I_Q}{2}} = \frac{g_m}{2} \qquad (11.72)$$

where g_m is each transistor's transconductance.

6.2) DC Transfer characteristics (Cont)

- The slope of i_{D2} characteristic curve at $v_d = 0$ is the same, except negative.

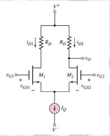
Note:

- Similar to BJT diff-amp, the differential-mode **input voltage must be held within a small range of voltages so as to remain linear**.
- However, the $v_d(\max)$ for the MOSFET diff-amp is much larger than $v_d(\max)$ for the BJT diff-amp. **Why?** Because the gain of the MOSFET diff-amp is much smaller than the gain of the bipolar (BJT) diff-amp.

Next

6.3) AC equivalent circuit

- **Figure 11.22** is AC equivalent circuit of diff-amp configuration, showing only diff voltage and signal currents as a function g_m. Assume output resistance looking into the current source is infinite.



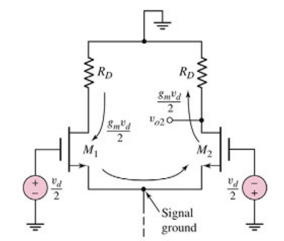


Figure 11.22: AC equivalent circuit for MOSFET differential amplifier.

6.3) AC equivalent circuit (Cont)

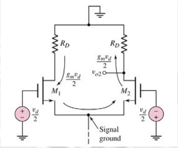


Figure 11.22: AC equivalent circuit for MOSFET differential amplifier.

- Using this equivalent circuit, the **One-sided output** voltage at v_{o2} is

$$v_{o2} \equiv v_o = +\frac{g_m v_d}{2} R_D \qquad (11.73)$$

- Then the differential voltage gain is

$$A_d = \frac{v_o}{v_d} = \frac{g_m R_D}{2} = \sqrt{\frac{K_n I_Q}{2}}.R_D \qquad (11.74)$$

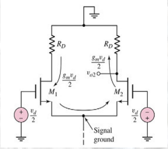
6.4) Differential- and Common-Mode Input Impedances

- At low frequencies, input impedance of a MOSFET is essentially infinite.

➔ This means that both the differential- and common-mode input resistances of a MOSFET diff-amp are **infinite**.

6.5) Small-signal Equivalent Circuit Analysis

- Can determine basic relationships for differential-mode gain, common-mode gain, and ***CMRR*** from an analysis of **the small-signal equivalent circuit**.



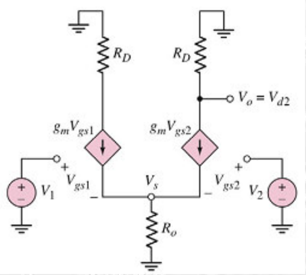


Fig 11.23: Small-signal equivalent circuit, MOSFET differential amplifier.

6.5) Small-signal Equivalent Circuit Analysis (Cont)

- Assume that transistors are matched, with $\lambda = 0$ for each transistor, and that constant-current source is represented by a **finite output resistance** ($R_o \neq \infty$).
- Two transistors are biased at the same quiescent current, and $g_{m1} = g_{m2} \equiv g_m$
- KCL equation at **node** V_s

$$g_m V_{gs1} + g_m V_{gs2} = \frac{V_s}{R_o}$$

From the circuit, $V_{gs1} = V_1 - V_s$ and $V_{gs2} = V_2 - V_s$

Then, $$g_m(V_1 + V_2 - 2V_s) = \frac{V_s}{R_o} \qquad (11.76)$$

6.5) Small-signal Equivalent Circuit Analysis (Cont)

- Solving for V_s

$$V_s = \frac{V_1 + V_2}{2 + \dfrac{1}{g_m R_o}} \qquad (11.77)$$

- For a **one-sided output** at the drain of M_2

$$V_o = V_{d2} = -(g_m V_{gs2})R_D = -(g_m R_D)(V_2 - V_s)$$

- Substitute (11.77) and rearranging terms yields

$$V_o = -g_m R_D \left[\frac{V_2\left(1 + \dfrac{1}{g_m R_o}\right) - V_1}{2 + \dfrac{1}{g_m R_o}} \right] \qquad (11.79)$$

6.5) Small-signal Equivalent Circuit Analysis (Cont)

- Based on relationships between input voltages V_1 and V_2 and differential- and common-mode voltages, as given by **Equation (11.29)**, Equation (11.79) can be written

$$V_o = \frac{g_m R_D}{2} V_d - \frac{g_m R_D}{1 + 2 g_m R_o} V_{cm} \qquad (11.80)$$

- The output voltage, in general form, is

$$V_o = A_d V_d + A_{cm} V_{cm}$$

- The transconductance g_m of the MOSFET is

$$g_m = 2\sqrt{K_n I_{DQ}} = \sqrt{2 K_n I_Q}$$

$$V_1 = V_{cm} + \frac{V_d}{2}$$

$$V_2 = V_{cm} - \frac{V_d}{2}$$

6.5) Small-signal Equivalent Circuit Analysis (Cont)

6.5.1) Differential- and Common-mode Gains

- Therefore:

➔ **Differential-mode gain** is

$$A_d = \frac{g_m R_D}{2} = \sqrt{2 K_n I_Q}\left(\frac{R_D}{2}\right) = \sqrt{\frac{K_n I_Q}{2}}.R_D \qquad (11.82(a))$$

➔ **Common-mode gain** is

$$A_{cm} = \frac{-g_m R_D}{1 + 2 g_m R_o} = \frac{-\sqrt{2 K_n I_Q}.R_D}{1 + 2\sqrt{2 K_n I_Q}.R_o} \qquad (11.82(b))$$

For an ideal current source, $R_O = \infty$, so $A_{cm} = 0$.

6.5) Small-signal Equivalent Circuit Analysis (Cont)

6.5.2) Common-Mode Rejection Ratio

➔ **Common-mode rejection ratio ($CMRR$)** is

$$CMRR = \frac{1}{2}\left[1 + 2\sqrt{2 K_n I_Q}.R_o\right] \qquad (11.83)$$

This demonstrate that the $CMRR$ for the MOSFET diff-amp is also **a strong function of the output resistance** of the constant-current source.

→ The value of $CMRR$ can be increased by increasing the output resistance of the current source. The increase can be accomplished by using a more sophisticated current source circuit, such as **the MOSFET cascode current mirror**.

6.5) Small-signal Equivalent Circuit Analysis (Cont)

Example 11.9

Objective:
Determine the differential-mode voltage gain, common-mode voltage gain, and CMRR for a MOSFET diff-amp.

Consider a MOSFET diff-amp with the configuration in **Figure 11.20**. Assume the same transistor **parameters as given in Example 11.8 except** assume $\lambda = 0.01\ \text{V}^{-1}$ for M_4

6.5) Small-signal Equivalent Circuit Analysis (Cont)

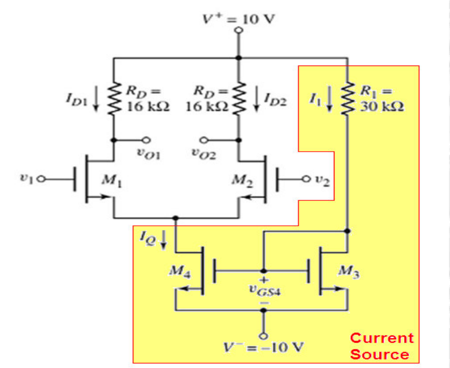


Figure 11.20: MOSFET diff amp for Example 11.8

6.5) Small-signal Equivalent Circuit Analysis (Cont)

Example 11.9 (Cont)

Solution:
From Example 11.8, bias current is I_Q **= 0.587 mA**.

The output resistance of the current source is then

R_O = 1 / (λI_Q) = 1/(0.01x0.587m) = **170 kΩ**

The differential-mode voltage gain is

$$A_d = \sqrt{\frac{K_n I_Q}{2}}.R_D = \sqrt{\frac{(1)(0.587\text{m})}{2}}.(16\text{k})$$

→ A_d **= 8.67**

6.5) Small-signal Equivalent Circuit Analysis (Cont)

Example 11.9 (Cont)

Solution: (Cont)

and the common-mode voltage gain is

$$A_{cm} = \frac{-\sqrt{2K_n I_Q}.R_D}{1+2\sqrt{2K_n I_Q}.R_o} = \frac{-\sqrt{2(1)(0.587\text{m})}.(16\text{k})}{1+2\sqrt{2(1)(0.587\text{m})}.(170\text{k})}$$

→ A_{cm} **= -0.0469**

The common-mode rejection ratio is then

$CMRR_{dB}$ **= 20log$_{10}$(8.67/0.0469) = 45.3 dB**

6.6) Two-Sided Output

- For an ideal MOSFET op-amp, with the **two-sided output** defined as with $V_o = V_{d2} - V_{d1}$, then

$$V_o = g_m R_D (V_{gs1} - V_{gs2}) = g_m R_D (V_1 - V_2)$$

With: $V_1 = V_{cm} + V_d / 2$ and $V_2 = V_{cm} - V_d / 2$

➔ **Differential-mode voltage gain** is
$A_d = g_m R_D$ (11.84(a))

➔**Common-mode voltage gain** is
$A_{cm} = 0$ (11.84(b))

$A_{cm} = 0$ is a consequence of using matched devices and elements in the diff-amp circuit.

6.6) Two-Sided Output (Cont)

6.6.1) Effect of R_D and g_m mismatches

	R_D mismatch $R_{D1} = R_D + \Delta R_D$ $R_{D2} = R_D - \Delta R_D$ $V_O = g_m(R_{D1}V_{gs1} - R_{D2}V_{gs2})$	**g_m mismatch** $g_{m1} = g_m + \Delta g_m$ $g_{m2} = g_m - \Delta g_m$ $V_O = R_D(g_{m1}V_{gs1} - g_{m2}V_{gs2})$
A_d	$g_m R_D$	$g_m R_D$
A_{cm}	$\cong \frac{\Delta R_D}{R_O}$	$\frac{R_D(2\Delta g_m)}{1+2R_O g_m}$
CMRR = $\left\lvert\frac{A_d}{A_{cm}}\right\rvert$	$\frac{g_m R_O}{(\Delta R_D / R_D)}$	$\frac{1+2R_O g_m}{2(\Delta g_m / g_m)}$

Note: CMRR of mismatched elements in MOSFET diff-amp **is identical** with the results of mismatched elements in the BJT diff-amp.

Next

Larger circuits

6.1) Basic MOSFET Differential Pair

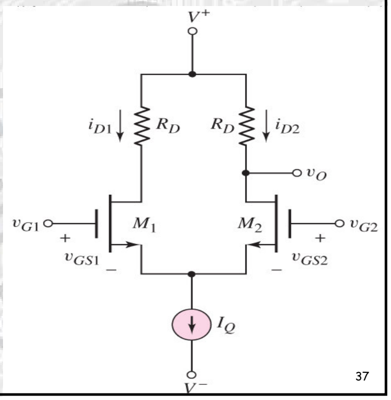


Figure 11.19: Basic MOSFET differential-pair configuration.

6.1) Basic MOSFET Differential Pair (Cont)

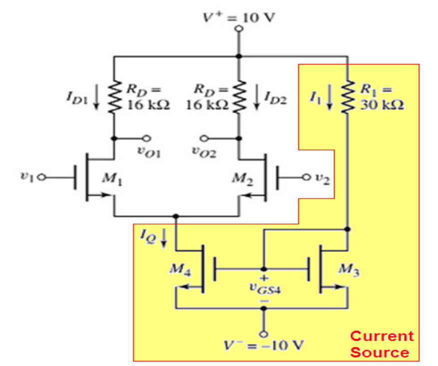


Figure 11.20: MOSFET diff amp for Example 11.8

6.2) DC Transfer characteristics (Cont)

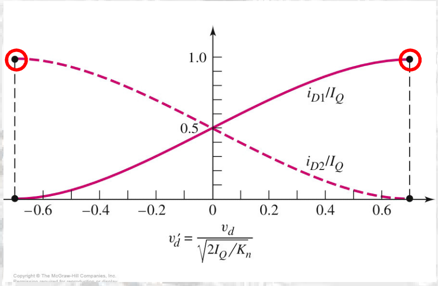


Fig 11.21: Normalized dc transfer characteristics for MOSFET differential amp.

➔ **This occurs when** $|v_d|_{\max} = \sqrt{I_Q / K_n}$ (11.70)

6.3) AC equivalent circuit

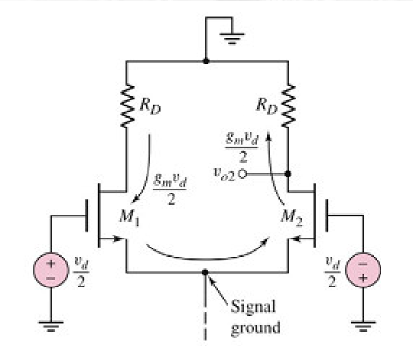


Figure 11.22: AC equivalent circuit for MOSFET diff-amp.

6.5) Small-signal Equivalent Circuit Analysis

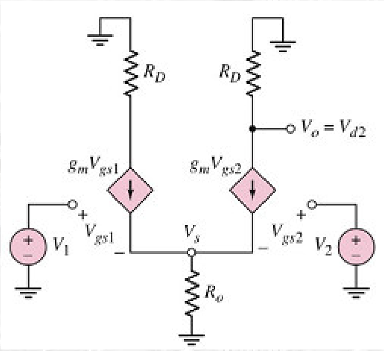


Fig 11.23: Small-signal equivalent circuit, MOSFET diff-amp.