(8)
Gain Stage and Simple Output Stage

Reference: Neamen, Chapter 11

Learning Outcome

Able to:

- Analyze an example of a gain stage and output stage of a multistage amplifier.

8.0) Introduction

- In virtually all operational amplifiers (**op-amps**), there are **3 stages**:

1) First stage is **Input Stage** ➔ Diff-amp with active load → to amplify difference between input signals v_1 and v_2.
2) Second stage is **Gain Stage** ➔ Darlington pair → to provide additional gain.
3) Third stage is **Output Stage** ➔ Emitter follower → to minimize loading effect on output signal.

8.1) Darlington Pair and Simple Emitter-Follower Output

- **Figure 11.46** shows a **BJT diff-amp** with a **3-transistor active load**, a **Darlington pair** connected to the diff-amp output, and a simple **emitter-follower output stage**.
- Diff-pair transistors (Q_1 and Q_2) are biased with a **Widlar current source** at a bias current I_Q.
- For the diff-amp currents **to be balanced**:

$$I_O = I_{B5} = \frac{I_Q}{\beta(1+\beta)} \qquad (11.118)$$

8.1) Darlington Pair and Simple Emitter-Follower Output (Cont)

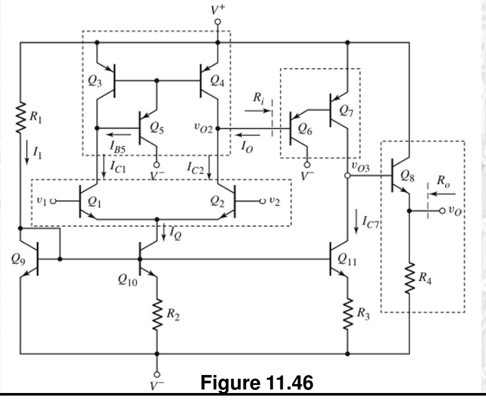


Figure 11.46

8.1) Darlington Pair and Simple Emitter-Follower Output (Cont)

- From the figure, can be seen

$$I_O = \frac{I_{E6}}{(1+\beta)} = \frac{I_{C7}}{\beta(1+\beta)} \qquad (11.119)$$

- In order for $I_O = I_{B5}$, require that $I_{C7} = I_Q$ ➔ Means that emitter resistors of Q_{10} and Q_{11} should have same value (i.e. $R_2 = R_3$).
- Q_{11} also **acts as an active load** for Darlington pair gain stage.
- Q_8 and R_4 form the simple emitter-follower output stage → minimizes loading effects because its output resistance is small.

8.1) Darlington Pair and Simple Emitter-Follower Output (Cont)

- **Ideally:**
  - ❑ When diff-amp input is a pure common-mode signal, the output $v_o = 0$.
  - ❑ The combination of Q_7 and Q_{11} allows dc level to shift.
  - ❑ By slightly changing bias current I_{C7} ➔ V_{EC7} and V_{CE11} can be varied such that $v_o = 0$.
  - ❑ This small variation in I_{C7} will not significantly change the balance between I_O and I_{B5}.

8.2) Darlington Pair: Input Impedance

- The input resistance of Darlington pair (Q_6 and Q_7) determines the loading effect on basic diff-amp.
- The gain of Darlington pair affects the overall gain of the op-amp circuit, and the output resistance of the emitter follower determines any loading effects on the output signal.
- **Figure 11.47(a)** is the **ac equivalent circuit** of the Darlington pair, where R_{L7} is the effective resistance connected between collector of Q_7 and signal ground.
- **Figure 11.47(b)** is the **simple hybrid-π model** of the Darlington pair Q_6 and Q_7 turned upside down.

8.2) Darlington Pair: Input Impedance (Cont)

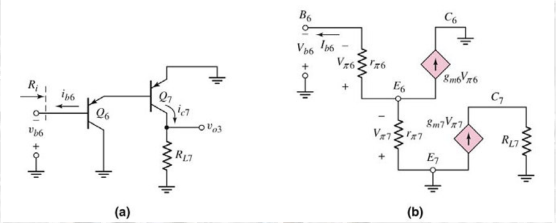


Figure 11.47: The Darlington pair's
(a) ac equivalent circuit, and
(b) small-signal equivalent circuit

8.2) Darlington Pair: Input Impedance (Cont)

- Writing a KVL equation around the B-E loop of Q_6 and Q_7 can obtain

$$V_{b6} = V_{\pi 6} + V_{\pi 7} \quad (11.120)$$

Also $$V_{\pi 6} = I_{b6} r_{\pi 6} \quad (11.121)$$

- The KCL equation at node E_6 is

$$\frac{V_{\pi 7}}{r_{\pi 7}} = \frac{V_{\pi 6}}{r_{\pi 6}} + g_{m6} V_{\pi 6}$$

or $$V_{\pi 7} = r_{\pi 7}\left[\frac{(1+\beta)}{r_{\pi 6}}\right] V_{\pi 6} = r_{\pi 7}(1+\beta) I_{b6} \quad (11.122(b))$$

where $r_{\pi 6} g_{m6} = \beta$

8.2) Darlington Pair: Input Impedance (Cont)

- Substitute (11.122(b)) and (11.121) into (11.120) to obtain

$$V_{b6} = I_{b6} r_{\pi 6} + r_{\pi 7}(1+\beta) I_{b6} \quad (11.123)$$

- The input resistance is therefore

$$R_i = \frac{V_{b6}}{I_{b6}} = r_{\pi 6} + r_{\pi 7}(1+\beta) \quad (11.124)$$

- Assuming $I_{C7} = I_Q$, the **hybrid-π** parameters are

$$r_{\pi 7} = \frac{\beta V_T}{I_{C7}} = \frac{\beta V_T}{I_Q} \quad (11.125(a))$$

$$r_{\pi 6} = \frac{\beta V_T}{I_{C6}} = \frac{(1+\beta)\beta V_T}{I_Q} \quad (11.125(b))$$

8.2) Darlington Pair: Input Impedance (Cont)

- Combining (11.125(a)), (11.125(b)), and (11.124) yields an expression for **input resistance**, as follows:

$$R_i = \frac{(1+\beta)\beta V_T}{I_Q} + \frac{(1+\beta)\beta V_T}{I_Q} = \frac{2(1+\beta)\beta V_T}{I_Q} \quad (11.126)$$

8.3) Darlington Pair: Voltage Gain

- To determine **small-signal voltage gain** of the Darlington pair, from **Fig 11.47(b)** can be obtained

$$v_{o3} = i_{c7}R_{L7} = (\beta i_{b7})R_{L7} = \beta(1+\beta)i_{b6}R_{L7}$$

$$i_{b6} = \frac{v_{b6}}{R_i}$$

Voltage Gain: $$A_v = \frac{v_{o3}}{v_{b6}} = \frac{\beta(1+\beta)R_{L7}}{R_i}$$

Thus, $$A_v = \frac{\beta(1+\beta)R_{L7}}{\frac{2(1+\beta)\beta V_T}{I_Q}} = \left(\frac{I_Q}{2V_T}\right)R_{L7} \qquad (11.130)$$

8.3) Darlington Pair: Voltage Gain (Cont)

- From **Figure 11.46**, can see that R_{L7} is **the parallel combination** of the resistance looking into collector of Q_{11} and the resistance looking into base of Q_8.
- Resistance looking into collector of Q_{11} is

$$R_{c11} = r_{o11}\left(1 + g_{m11}R_E^{'}\right) \qquad (11.131)$$

where $$R_E^{'} = r_{\pi 11} \| R_3$$

- Resistance looking into base of Q_8 is

$$R_{b8} = r_{\pi 8} + (1+\beta)R_4 \qquad (11.132)$$

➔ Since R_{c11} and R_{b8} are large, then the effective resistance R_{L7} is also large.

8.3) Darlington Pair: Voltage Gain (Cont)

Example 11.13

Objective:
Calculate the input resistance and the small-signal voltage gain of a Darlington pair.

Consider the circuit shown in **Figure 11.46**, with parameters $I_{C7} = I_Q =$ **0.2 mA**, $I_{C8} =$ **1 mA**, $R_4 =$ **10 kΩ**, and $R_3 =$ **0.2 kΩ**. Assume $\beta = 100$ for all transistors, and the Early voltage for Q_{11} is **100 V**.

8.3) Darlington Pair: Voltage Gain (Cont)

Example 11.13 (Cont)

Solution: The input resistance, given by Equation (11.126), is

$$R_i = \frac{2(1+\beta)\beta V_T}{I_Q} = \frac{2(101)(100)(0.026)}{0.2\text{m}} \Rightarrow 2.63M\Omega$$

The small-signal voltage gain is a function of R_{L7}, which in turn is a function of R_{c11} and R_{b8}. We can find that

$r_{\pi 11} = \beta\, V_T / I_Q = (100)(0.026)/(0.2\text{m}) = \mathbf{13\ k\Omega}$

8.3) Darlington Pair: Voltage Gain (Cont)

Example 11.13 (Cont)

such that

$R'_E = 13\text{k} \| 0.2\text{k} = \mathbf{0.197\ k\Omega}$

Also

$g_{m11} = I_Q / V_T = 0.2\text{m}/0.026 = \mathbf{7.69\ mA/V}$

and

$r_{o11} = V_A / I_Q = 100/0.2\text{m} = \mathbf{500\ k\Omega}$

Therefore,

$R_{c11} = r_{o11}\,(1 + g_{m11}R'_E) = \mathbf{1.26\ M\Omega}$

8.3) Darlington Pair: Voltage Gain (Cont)

Example 11.13 (Cont)

We can determine that

$r_{\pi 8} = \beta\, V_T / I_{C8} = (100)(0.026)/(1\text{m}) = \mathbf{2.6\ k\Omega}$

Then

$R_{b8} = r_{\pi 8} + (1+\beta)R_4 = 2.6\text{k}+(101)(10\text{k}) = \mathbf{1.01\ M\Omega}$

Consequently, resistance R_{L7} is

$R_{L7} = R_{c11} \| R_{b8} = 1.26\text{M} \| 1.01\text{M} = \mathbf{0.561\ M\Omega}$

Finally, from Equation (11.130), the small-signal voltage gain is

$A_v = (I_Q / 2V_T)\, R_{L7} = \mathbf{2158}$

8.4) Emitter Follower: Output Resistance

- From **Figure 11.46**, the output resistance of the emitter follower Q_8 is

$$R_o = R_4 \left\| \left(\frac{r_{\pi 8} + Z}{(1+\beta)} \right) \right. \quad (11.133)$$

where Z is the equivalent impedance, or resistance, in the base of Q_8.

- In this case, $Z = R_{c11} \,||\, R_{c7}$, where R_{c7} is resistance looking into the collector of Q_7
- The factor **($1+\beta$)** in the denominator makes output resistance of the emitter follower **normally small**.

8.4) Emitter Follower: Output Resistance (Cont)

Example 11.14

Objective:
Calculate the output resistance of the circuit in **Figure 11.46**.

Consider the same circuit and transistor parameters described in **Example 11.13**. Assume the Early voltage of Q_7 is 100 V.

8.4) Emitter Follower: Output Resistance (Cont)

Example 11.14 (Cont)

Solution: From **Example 11.13**, we have that $R_{c11} = 1.26\text{ M}\Omega$ and $r_{\pi 8} = 2.6\text{ k}\Omega$.

We can determine that

$$R_{c7} = V_A / I_Q = 100/0.2\text{m} = \mathbf{500\ k\Omega}$$

Then,

$$Z = R_{c11} \| R_{c7} = 1.26\text{M} \| 500\text{k} = \mathbf{358\ k\Omega}$$

Therefore

$$R_o = R_4 \left\| \left(\frac{r_{\pi 8} + Z}{(1+\beta)} \right) = 10k \right\| \left(\frac{2.6\text{k} + 358\text{k}}{(101)} \right) = 2.63\text{k}\Omega$$

8.5) Overall Gain of the Op-amp circuit

- Input stage: Diff-amp with active load → $A_{V1} \approx 10^3$
- Gain stage: Darlington pair → $A_{V2} \approx 10^3$
- Output stage: Emitter-follower → $A_{V3} \approx 1$
- **Overall Gain:** $A_V = (A_{V1}).(A_{V2}).(A_{V3}) \approx 10^6$

⇨ **10^6 is the typical value** for low-frequency, open-loop gain of the op-amp circuits

Larger circuits

8.1) Darlington Pair and Simple Emitter-Follower Output (Cont)

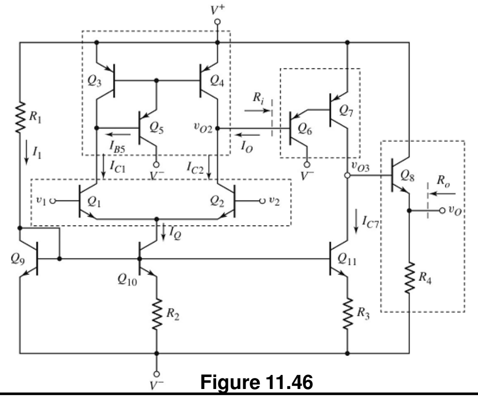
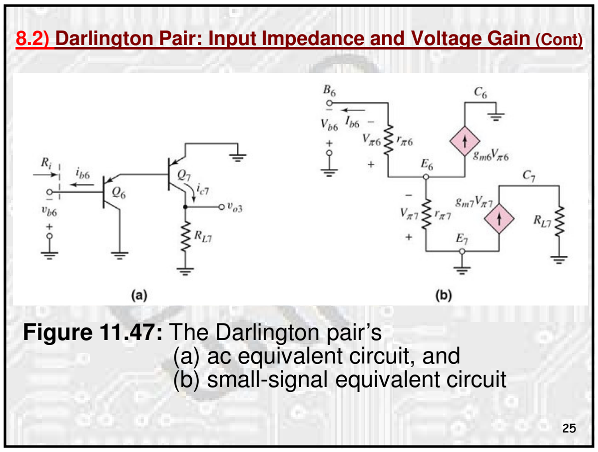


Figure 11.46



8.2) Darlington Pair: Input Impedance and Voltage Gain (Cont)

Figure 11.47: The Darlington pair's
(a) ac equivalent circuit, and
(b) small-signal equivalent circuit