Name:

Student ID Number:

Section: **01/02/03/04/05 A/B**

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UNIVERSITI TENAGA NASIONAL

College of Engineering

Department of Electronics and Communication Engineering

Test 1

SEMESTER 1, ACADEMIC YEAR 2012/2013

Subject Code	:	**EEEB273**
Course Title	:	**Electronics Analysis & Design II**
Date	:	**29 June 2012**
Time Allowed	:	**1.5 hours**

Instructions to the candidates:

1. Write your Name and Student ID number. Circle your section number.
2. Write all your answers using pen. **DO NOT USE PENCIL** except for the diagram.
3. **ANSWER ALL QUESTIONS.**
4. **WRITE YOUR ANSWER ON THIS QUESTION PAPER.**
5. For BJT, use V_T **= 26 mV** where appropriate.
6. Use at least **4 significant numbers** in all calculations.

NOTE: DO NOT OPEN THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO.

☺ **GOOD LUCK!** ☺

Question No.	1	2	3	Total
Marks				

<u>Question 1</u> **[40 marks]**

a) **Explain clearly** why are currents Ic_1 and Ic_2 in the **basic two-transistor BJT** current source similar? [5 marks]

b) **The values** of β for the transistors in **Figure 1.1** are **very large**.

i) If Q_1 in the **Figure 1.1** is **diode-connected**, as shown in **Figure 1.2**, to provide constant current $I_{REF} = I_1 = 0.5$ **mA**, **determine** the collector currents in the other transistors, **i.e.** I_2 **and** I_3. [6 marks]

ii) **Find** I_1 and I_3 if Q_2 **is diode-connected** to provide $I_{REF} = I_2 = 0.5$ **mA**. [2 marks]

iii) **Find** I_1 and I_2 if Q_3 **is diode-connected** to provide $I_{REF} = I_3 = 0.5$ **mA**. [2 marks]

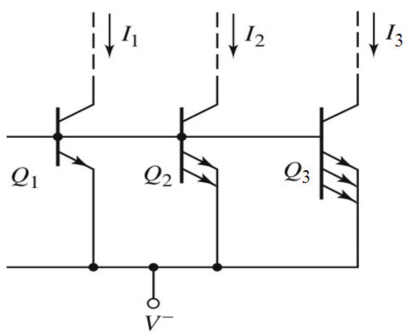

Figure 1.1

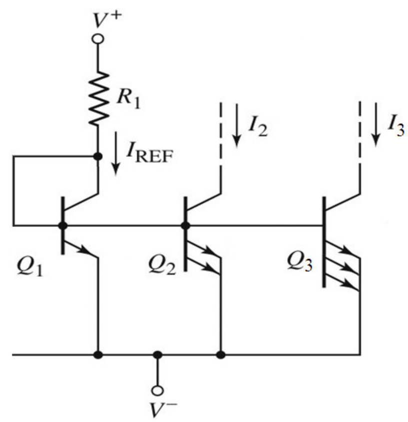

Figure 1.2

<u>Answers for Question 1</u>

Q1(a)

- Transistors Q_1 and Q_2 are matched, i.e. $I_{S1} = I_{S2}$, as well as other parameters eg β, V_A and V_{EB}(on). [1.5]
- From the circuit, $V_{BE1} = V_{BE2}$ [1.5]
- From the equation $I_C = I_S$ (exp(V_{BE} / V_T)), [1]
  if Q_1 and Q_2 are matched and have the same V_{BE}, then $I_{C1} = I_{C2}$ [1]

c) **Figure 1.3** shows a **pnp current source** with transistor parameters β **= 50,** V_A **= 50 V** and V_{EB}**(on) = 0.7 V**.

i) **Find** R_1 such that I_{REF} **= 1.5 mA** and find the value of I_O. [7 marks]

ii) **What** is the maximum value of R_{C2} such that Q_2 remains in the **forward-active region**? Assume V_{EC2}**(min)** = V_{EB}**(on)**. [3 marks]

iii) **Find** the change in I_O, i.e. dI_O, if V_O **= 3.5 V**. [9 marks]

iv) **Calculate** the percentage change in I_O for **part iii)** above. [3 marks]

v) The circuit in **Figure 1.3 is modified** to include a resistor R_E at the emitter of Q_2. **Discuss** what will happen to the percentage change in I_O of the current source in this situation. [3 marks]

Answers for Question 1 (Cont.)

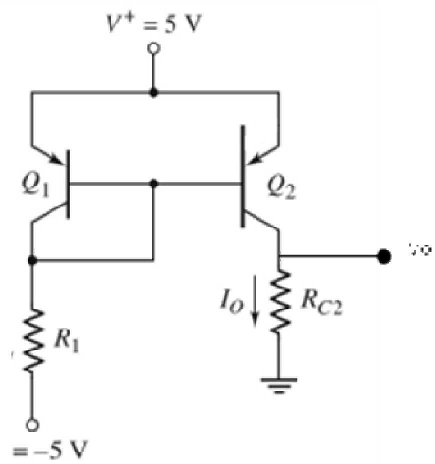

Figure 1.3

Q1(b)

i) $I_{REF} = I_1 = 0.5$ mA
From Figure 1.2,
$I_2 = 2\, I_{REF} = 2\, I_1$ [2]
$= 2\,(0.5m) = 1$ mA [1]
$I_3 = 3\, I_{REF} = 3\, I_1$ [2]
$= 3\,(0.5m) = 1.5$ mA [1]

ii) $I_{REF} = I_2 = 0.5$ mA
Similar with above,
$I_1 = I_2/2 = (0.5m)/2 = 0.25$ mA [1]
$I_3 = 3\, I_1 = 3(0.25m) = 0.75$ mA [1]

iii) $I_{REF} = I_3 = 0.5$ mA
Similar with above,
$I_1 = I_3/3 = (0.5m)/3 = 0.167$ mA [1]
$I_2 = 2\, I_1 = 2(0.167m) = 0.33$ mA [1]

Answers for Question 1 (Cont.)

Q1c(i)

$$I_{REF} = \frac{V^{+} - V_{EB}(\text{on}) - V^{-}}{R_1} = 1.5\text{mA} \quad [2]$$

$$R_1 = \frac{5 - 0.7 - (-5)}{1.5\text{m}} = 6.22\text{k}\Omega \quad [2]$$

$$I_O = \frac{I_{REF}}{1 + 2/\beta} = \frac{1.5\text{m}}{1 + 2/50} = 1.44\text{mA} \quad [3]$$

Q1c(ii)

$$R_2(\max) = \frac{V^{+} - V_{EC}(\min)}{I_O} \quad [2]$$

$$R_2(\max) = \frac{5 - 0.7}{1.44\text{mA}} = 2.986\text{k}\Omega \quad [1]$$

Q1c(iii)

dIo = dVo/Ro [2]

*Ro = ro*2 [1]
*ro*2 = V_A/*Io* = 50/(1.44m) = 34.72kΩ [2]

Vo(original) = V^+ - $\mathbf{V_{EC}(min)}$ = 5 - 0.7 = 4.3V [2]

So, *dVo* = 4.3 V - 3.5 V = 0.8 V [1]

So, *dIo* = 0.8 V/34.72 kΩ = 0.023 mA [1]

Q1c(iv)

% change in *Io* = *dIo/Io* x 100% [2]
= 0.023m/1.44m x 100% = 1.597% [1]

Q1c(v)

- The circuit becomes the Widlar current source. [0.5]
- Widlar's output impedance *Ro* is much larger, [0.5]
  i.e. *Ro*= *ro*2[1 +gm(rπ//R_E)]. [0.5]
- Hence, lowering the *dIo*. [0.5]
- So, the % change in *Io* will decrease, [0.5]
- thus, improving the performance of the current source in terms of stability of *Io*. [0.5]

<u>Question 2</u> **[30 marks]**

The current equation for an **NMOS transistor** is given by:

$$i_D = K_n(v_{GS} - V_{TN})^2(1 + \lambda v_{DS})$$

where $$K_n = \frac{k_n^{'}}{2}\left(\frac{W}{L}\right)$$

Consider the **basic MOSFET two-transistor** current source in **Figure 2**. The circuit parameters are $\mathbf{V^+ = 3}$ **V,** $\mathbf{V^- = -3}$ **V**, and $\mathbf{I_{REF} = 120}$ **μA**; and the transistor parameters are $\mathbf{V_{TN} = 0.8}$ **V** and $\mathbf{k'_n =}$ **80 μA/V²**.

a) **Give** another name for the MOSFET **current source**. [1 mark]

b) For the basic **two-transistor NMOS** current source, **find** the relationship between I_O and I_{REF}. [5 marks]

c) **Find** V_{GS1}, V_{GS2}, V_{DS1}, and I_O at $\lambda = 0$ for the following transistor **aspect ratios**:

i) $(W/L)_1 = (W/L)_2 = 4.5$ [8 marks]

ii) $(W/L)_1 = 4.5$ and $(W/L)_2 = 2.25$ [8 marks]

d) For $(W/L)_1 = 4.5$, $(W/L)_2 = 2.25$ and $\lambda = 0.02$ V^{-1}, **calculate** the change in I_O if V_{DS2} changes by **0.75 Volts**. [8 marks]

<u>Answers for Question 2</u>

Q2(a)
Current mirror [1]

Q2(b)

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN1})^2(1 + \lambda_1 V_{DS1})$$ [1]

$$I_O = K_{n2}(V_{GS2} - V_{TN2})^2(1 + \lambda_2 V_{DS2})$$ [1]

$$\frac{I_{REF}}{I_O} = \frac{K_{n1}(V_{GS1} - V_{TN1})^2(1 + \lambda_1 V_{DS1})}{K_{n2}(V_{GS2} - V_{TN2})^2(1 + \lambda_2 V_{DS2})} = \frac{(W/L)_1(1 + \lambda_1 V_{DS1})}{(W/L)_2(1 + \lambda_2 V_{DS2})}$$ [2]

$$I_O = I_{REF}\left[\frac{(W/L)_2(1 + \lambda_2 V_{DS2})}{(W/L)_1(1 + \lambda_1 V_{DS1})}\right]$$ [1]

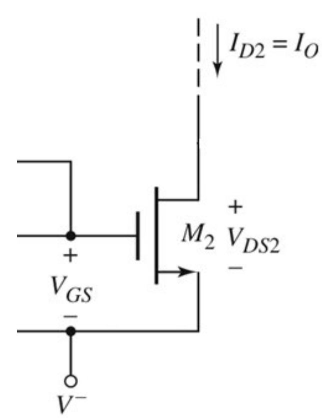

Figure 2

Answers for Question 2 (Cont.)

Q2(c)(i)

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN1})^2$$ [1]

$$V_{GS1} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}}$$ [1]

$$V_{GS1} = 0.8 + \sqrt{\frac{120\mu}{\left(80\mu/2\right)(4.5)}} = 0.8 + 0.8165 = 1.6165\text{V}$$ [1.5]

$$V_{GS2} = V_{GS1} = V_{DS1} = 1.6165\text{V}$$ [1.5]

$$I_O = I_{REF}\left[\frac{(W/L)_2(1+\lambda_2 V_{DS2})}{(W/L)_1(1+\lambda_1 V_{DS1})}\right]$$ [2]

$$I_O = I_{REF} = 120\mu\text{A}$$ [1]

Q2(c)(ii)

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN1})^2$$ [1]

$$V_{GS1} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}}$$ [1]

$$V_{GS1} = 0.8 + \sqrt{\frac{120\mu}{\left(80\mu/2\right)(4.5)}} = 0.8 + 0.8165 = 1.6165\text{V}$$ [1.5]

$$V_{GS2} = V_{GS1} = V_{DS1} = 1.6165\text{V}$$ [1.5]

$$I_O = I_{REF}\left[\frac{(W/L)_2}{(W/L)_1}\right]$$ [1.5]

$$I_O = 120\mu\left(\frac{2.25}{4.5}\right) = 60\mu\text{A}$$ [1.5]

Q2(d)

$$V_{GS2} = V_{GS1} = 1.6165\text{V}$$ [1]

$$I_O = \frac{k_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2 = \frac{80\mu}{2}(2.25)(0.8165)^2 = 60\mu\text{A}$$ [2.5]

$$r_{O2} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(60\mu)} = 833\text{k}\Omega$$ [1.5]

$$R_O = r_{O2}$$ [1]

$$\frac{dI_O}{dV_{DS2}} = \frac{1}{R_O} \Rightarrow dI_O = \frac{(dV_{DS2})}{(R_O)}$$ [1]

$$dI_O = \frac{(0.75)}{(833\text{k})} = 0.9003\mu\text{A}$$ [1]

Question 3 **[30 marks]**

a) **Figure 3.1** shows a basic BJT differential pair.
Assume that $\boldsymbol{Q_1}$ and $\boldsymbol{Q_2}$ are matched pair and operating at the same temperature.

i) By defining $v_d = v_{BE1} - v_{BE2}$

Show that $i_{C1} = \dfrac{I_Q}{1+e^{-v_d/V_T}}$ and $i_{C2} = \dfrac{I_Q}{1+e^{+v_d/V_T}}$

[10 marks]

ii) **What** happen when differential-mode input voltage ($\boldsymbol{v_d}$) is zero? **Show** how the answer is obtained and **explain** your answer.

[4 marks]

Answers for Question 3

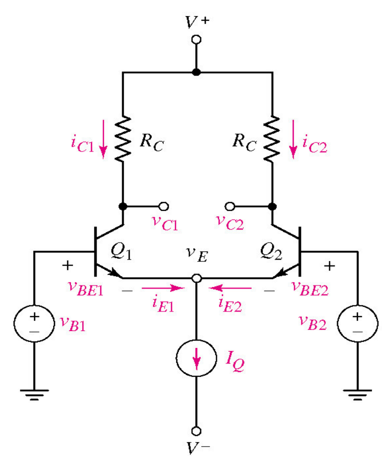

Figure 3.1

b) **Figure 3.2** shows the **small-signal equivalent circuit** for basic BJT differential pair of **Figure 3.1**, where v_{B1} and v_{B2} in the **Figure 3.1** are represented by 2 input signals V_{b1} and V_{b2} respectively, and their input signal resistors R_B in the **Figure 3.2**.

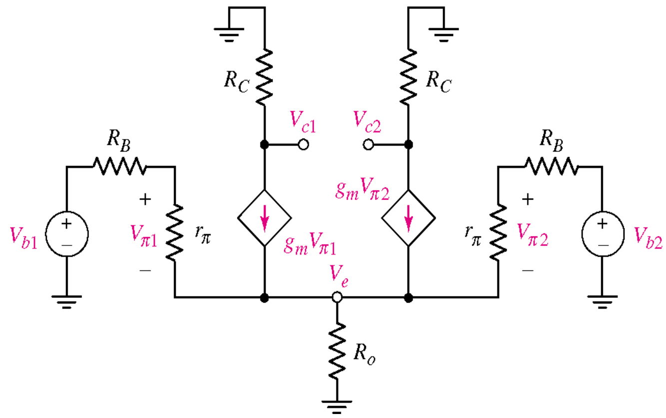

Figure 3.2

With small-signal analysis, it can be found that **one-sided output** (V_o) taken at collector of Q_2 (i.e. V_{c2}) is given by:

$$V_o = \frac{-\beta R_c}{r_\pi + R_B}\left\{\frac{V_{b2}\left[1+\frac{r_\pi + R_B}{(1+\beta)R_o}\right]-V_{b1}}{2+\frac{r_\pi + R_B}{(1+\beta)R_o}}\right\}$$

If differential-mode input is $V_d = v_{BE1} - v_{BE2} = (V_{b1} - V_e) - (V_{b2} - V_e) = V_{b1} - V_{b2}$ and **an ideal constant-current source** is used to bias the BJT differential pair, **show that** the differential-mode gain is

$$A_d = \frac{V_o}{V_d} = \frac{\beta R_c}{2(r_\pi + R_B)}$$

[4 marks]

c) For the differential amplifier shown in **Figure 3.1**, given that $i_{E1} = 0.4$ **mA** and $R_C = 12$ **kΩ**. The transistor parameters are $\beta = 100$ and $V_A = \infty$. Assume the output resistance looking into the constant-current source is $R_o = 25$ **kΩ** and the input signal resistors R_B **are zero**. The common-mode gain is given by:

$$A_{cm} = -\frac{g_m R_C}{1+\frac{2(1+\beta)R_o}{r_\pi + R_B}}$$

Consider a **one-sided output** taken at v_{C2}. **Calculate**:

i) The differential-mode gain. [4 marks]

ii) The common-mode gain. [5 marks]

iii) The common-mode rejection ratio (***CMRR***). [3 marks]

Answers for Question 3 (Cont.)

Q3a(i)

$$i_{C1} = I_S e^{v_{BE1}/V_T}, \quad i_{C2} = I_S e^{v_{BE2}/V_T} \quad \boxed{1} \quad \boxed{1}$$

$$I_Q = i_{C1} + i_{C2} = I_S\left[e^{v_{BE1}/V_T} + e^{v_{BE2}/V_T}\right] \quad \boxed{1}$$

$$\frac{i_{C1}}{I_Q} = \frac{I_S e^{v_{BE1}/V_T}}{I_S\left[e^{v_{BE1}/V_T} + e^{v_{BE2}/V_T}\right]} = \frac{I_S(e^{v_{BE1}/V_T})/e^{v_{BE1}/V_T}}{I_S\left[e^{v_{BE1}/V_T} + e^{v_{BE2}/V_T}\right]/e^{v_{BE1}/V_T}} = \frac{1}{1+e^{(v_{BE2}-v_{BE1})/V_T}} \quad \boxed{2}$$

$$\frac{i_{C2}}{I_Q} = \frac{I_S e^{v_{BE2}/V_T}}{I_S\left[e^{v_{BE1}/V_T} + e^{v_{BE2}/V_T}\right]} = \frac{I_S(e^{v_{BE2}/V_T})/e^{v_{BE2}/V_T}}{I_S\left[e^{v_{BE1}/V_T} + e^{v_{BE2}/V_T}\right]/e^{v_{BE2}/V_T}} = \frac{1}{1+e^{-(v_{BE2}-v_{BE1})/V_T}} \quad \boxed{2}$$

$$v_{BE1} - v_{BE2} = v_d \quad \boxed{1}$$

$$\rightarrow i_{C1} = \frac{I_Q}{1+e^{(v_{BE2}-v_{BE1})/V_T}} = \frac{I_Q}{1+e^{-vd/V_T}} \quad \boxed{1}$$

$$\rightarrow i_{C2} = \frac{I_Q}{1+e^{-(v_{BE2}-v_{BE1})/V_T}} = \frac{I_Q}{1+e^{+vd/V_T}} \quad \boxed{1}$$

Q3a(ii)

$$i_{C1} = \frac{I_Q}{1+e^{-v_d/V_T}} = \frac{I_Q}{1+e^{-0/V_T}} = \frac{I_Q}{1+1} = \frac{I_Q}{2} \quad \boxed{1}$$

$$i_{C2} = \frac{I_Q}{1+e^{+v_d/V_T}} = \frac{I_Q}{1+e^{0/V_T}} = \frac{I_Q}{1+1} = \frac{I_Q}{2} \quad \boxed{1}$$

When $\boldsymbol{v_d = 0, i_{C1} = i_{C2} = I_Q/2}$ → current $\boldsymbol{I_Q}$ splits evenly between $\boldsymbol{i_{C1}}$ and $\boldsymbol{i_{C2}}$ $\boxed{1}$

$\boxed{1}$

Q3b

For ideal constant-current source $\boldsymbol{R_o = \infty}$ $\boxed{1}$

$$V_o = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{V_{b2}\left[1+\frac{r_\pi + R_B}{(1+\beta)R_o}\right] - V_{b1}}{2+\frac{r_\pi + R_B}{(1+\beta)R_o}}\right\}$$

$$R_o = \infty \rightarrow V_o = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{V_{b2}-V_{b1}}{2}\right\} = \frac{-\beta R_C}{r_\pi + R_B}\left\{\frac{-V_d}{2}\right\}$$

$$A_d = \frac{V_o}{V_d} = \frac{\beta R_C}{2(r_\pi + R_B)} \quad \boxed{1} \quad \boxed{1}$$

$\boxed{1}$

Q3c(i)

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$R_B = 0 \rightarrow A_d = \frac{\beta R_C}{2(r_\pi)}$$ [1]

Either:

$$g_m = \frac{\beta}{r_\pi} \rightarrow A_d = \frac{g_m R_C}{2}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{i_{E1}}{V_T} = \frac{0.4\text{m}}{0.026} = 15.385\text{mA/V}$$ [1, 0.5]

$$A_d = \frac{g_m R_C}{2} = \frac{(15.385\text{m})(12\text{k})}{2} = 92.3$$ [1, 0.5]

Or:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{\beta V_T}{i_{E1}} = \frac{(100)(0.026)}{0.4\text{m}} = 6.5\text{k}\Omega$$ [1, 0.5]

$$A_d = \frac{\beta R_C}{2(r_\pi)} = \frac{(100)(12\text{k})}{2(6.5\text{k})} = 92.3$$ [0.5, 0.5, 0.5]

Q3c(ii)

$$A_{cm} = -\frac{g_m R_C}{1 + \frac{2(1+\beta)R_o}{r_\pi + R_B}}$$

$$R_B = 0 \rightarrow A_{cm} = -\frac{g_m R_C}{1 + \frac{2(1+\beta)R_o}{r_\pi}}$$ [1]

$$g_m = \frac{I_{CQ}}{V_T} = \frac{i_{E1}}{V_T} = \frac{0.4\text{m}}{0.026} = 15.385\text{mA/V}$$ [1]

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{\beta V_T}{i_{E1}} = \frac{(100)(0.026)}{0.4\text{m}} = 6.5\text{k}\Omega$$ [1]

$$A_{cm} = -\frac{g_m R_C}{1 + \frac{2(1+\beta)R_o}{r_\pi}} = -\frac{(15.385\text{m})(12\text{k})}{1 + \frac{2(1+100)(25\text{k})}{6.5\text{k}}} = -0.237$$ [2]

Q3c(iii)

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \left|\frac{92.3}{-0.237}\right| = 389$$

[1] [1] [1]

Appendix: BASIC FORMULA

BJT

$$i_C = I_S e^{v_{BE}/V_T} \text{ ;npn}$$

$$i_C = I_S e^{v_{EB}/V_T} \text{ ;pnp}$$

$$i_C = \alpha i_E = \beta i_B$$

$$i_E = i_B + i_C$$

$$\alpha = \frac{\beta}{\beta + 1}$$

;Small signal

$$\beta = g_m r_\pi$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

$$g_m = \frac{I_{CQ}}{V_T}$$

$$r_o = \frac{V_A}{I_{CQ}}$$

MOSFET

;N – MOSFET

$$v_{DS}(\text{sat}) = v_{GS} - V_{TN}$$

$$i_D = K_n[v_{GS} - V_{TN}]^2$$

$$K_n = \frac{k_n'}{2} \cdot \frac{W}{L}$$

;P – MOSFET

$$v_{SD}(\text{sat}) = v_{SG} + V_{TP}$$

$$i_D = K_p[v_{SG} + V_{TP}]^2$$

$$K_p = \frac{k_p'}{2} \cdot \frac{W}{L}$$

;Small signal

$$g_m = 2K_n\left(V_{GSQ} - V_{TN}\right) = 2\sqrt{K_n I_{DQ}}$$

$$r_o \cong \frac{1}{\lambda I_{DQ}}$$