UNIVERSITI TENAGA NASIONAL

College of Information Technology

BACHELOR OF INFORMATION TECHNOLOGY (HONS.)

BACHELOR OF COMPUTER SCIENCE (HONS.)

FINAL EXAMINATION

SEMESTER II 2012/2013

DISCRETE STRUCTURES

(CSNB143)

JANUARY 2013

Time allowed: 2 hours 30 minutes + 10 minutes for reading

INSTRUCTIONS TO CANDIDATES

1. There are **TWO (2) SECTIONS** to this paper: Section A and Section B
2. Answer **ALL** questions in **Section A** and **Section B**.
3. The total marks for this exam is 100 marks.
4. Answer **ALL** the questions in the answer booklet provided.
5. You are allowed to use non-programmable calculators during the exam.

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE INSTRUCTED TO DO SO

THIS QUESTION PAPER CONSISTS OF 8 PRINTED PAGES INCLUDING THIS PAGE

SECTION A: SHORT ANSWER QUESTIONS (10 QUESTIONS, 30 MARKS)

Instruction: Answer ALL of the following questions.

Question 1

(a) Let A = {ab, bb, bc}. In each of the following parts, indicate whether the string belongs to A^*. (Note: Answer **YES** or **NO**)

(i) abababbbbbc

[1 mark]

(ii) bbabbbbcbbc

[1 mark]

(b) Let A = {1, 2, 3, {4, 5, 6}}. Identify each of the following as **TRUE** or **FALSE**.

(i) $\{4, 5, 6\} \subseteq A$.

[1 mark]

(ii) $\{3\} \in A$

[1 mark]

(iii) $\{1, 2\} \subseteq A$

[1 mark]

Question 2

Let p, q, and r be the following statements:

p: *I will study Discrete Structures.*

q: *I will go for dinner.*

r: *I am happy.*

Write the statements below in terms of p, q, r, and logical connectives.

(a) If I am not happy, then I will go for dinner.

[2 marks]

(b) I am happy only if I will not study Discrete Structures and I will go for dinner.

[2 marks]

Question 3

Let the Universal set U = {a, b, c, d, e, f, g, h, i, j}, and let A and B be two sets where A = {a, b, c, d, f, g, i} and B = {a, c, f, g, h, i, j}. Compute the following:

(a) $A - B$

[1 mark]

(b) $B \oplus A$

[1 mark]

Question 4

(a) Let $A = \{a, b, c\}$. Find $P(A)$.

[2 marks]

(b) What is $|P(A)|$?

[1 mark]

Question 5

Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 3), (3, 2), (3, 3), (4, 2)}.

(a) Find the *symmetric closure* of R.

[1 mark]

(b) Find the *transitive closure* of R.

[3 marks]

Question 6

Determine if the relation R = {(1, 7), (2, 3), (2, 6), (4, 1), (4, 2), (4, 5), (5, 3)} is a tree on the set A = {1, 2, 3, 4, 5, 6, 7}. If it is a tree, what is the root? If it is not a tree, then make the least number of changes necessary to make it a tree and give the root.

[4 marks]

Question 7

Write a formula for the n^{th} term of the following sequence. Identify your formula as recursive or explicit.

1, -1, 1, -1, 1, -1 . . .

[2 marks]

Question 8

In a psychological experiment, a person must arrange a square, a cube, a triangle and a pentagon in a row. How many different arrangements are possible?

[1 mark]

Question 9

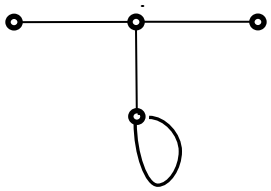
Draw a picture of the graph $G = (V, E, \gamma)$ where $V = \{a, b, c, d, e\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, and $\gamma(e_1) = \gamma(e_5) = \{a, c\}$, $\gamma(e_2) = \{a, d\}$, $\gamma(e_3) = \{e, c\}$, $\gamma(e_4) = \{b, c\}$ and $\gamma(e_6) = \{e, d\}$.

[2 marks]

Question 10

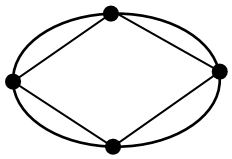
For the following graphs, tell whether the graph has an Euler circuit, an Euler path but no Euler circuit or neither. Give reasons for your choice.

(a)



[1.5 marks]

(b)



[1.5 marks]

SECTION B: PROBLEM-SOLVING (10 QUESTIONS, 70 MARKS)

<u>Instruction</u>: Answer ALL of the following questions.

Question 1

(a) Convert the *Hexadecimal* number $\mathbf{AC2_{16}}$ to its equivalent *Decimal* number.

[2 marks]

(b) Convert the *Octal* number $\mathbf{52_8}$ to its equivalent *Binary* number.

[2 marks]

(c) Evaluate the *Octal addition* $\mathbf{(35_8 + 64_8)}$.

[2 marks]

Question 2

Let **A** and **B** be two matrices as given below. Compute **AB**.

$$\mathbf{A} = \begin{matrix} 1 & 4 & 1 \\ 3 & 1 & -1 \\ 1 & 2 & 2 \end{matrix}, \quad \mathbf{B} = \begin{matrix} 1 & 2 \\ -3 & 1 \\ 2 & -2 \end{matrix}$$

[6 marks]

Question 3

Prove that $n < 2^n$ for all $n \geq 2$ using mathematical induction.

[6 marks]

Question 4

Four fair six-sided dice are tossed and the numbers showing on top are recorded.

(a) How many different record sequences are possible?

[2 marks]

(b) How many of the records in part (a) contain **EXACTLY** one *three*?

[3 marks]

(c) How many of the records in part (a) contain **EXACTLY** three *fours*?

[3 marks]

Question 5

A committee of 9 people with one person designated as chair of the committee is to be formed. How many different committees of this type can be chosen from a group of 12 people?

[5 marks]

Question 6

(a) Let $A = \{1, 2, 3, 4, 5\}$. Determine whether the relation R with matrix $\mathbf{M}_R$ as given below is *symmetric*, *asymmetric*, or *antisymmetric*. Briefly *explain* your answers.

$$\mathbf{M}_R = \begin{matrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

[6 marks]

(b) Let $A = \{1, 2, 3, 4\}$. Let

$R = \{(1, 1), (1, 2), (2, 3), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3)\}$

$S = \{(1, 1), (1, 4), (2, 3), (2, 4), (3, 1), (3, 3), (4, 4)\}$.

Compute or find $R \circ S$.

[4 marks]

Question 7

Show or prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cup C$.

[4 marks]

Question 8

Let $A = \{1, 2, 3, 4\}$ and R be a relation on A with matrix $\mathbf{M}_R$ as given below:

$$\mathbf{M}_R = \begin{matrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{matrix}$$

(a) Prove that R is a *partial order*.

[4 marks]

(b) Draw the *Hasse* diagram of R.

[4 marks]

Question 9

Let $A = B = C = \mathbb{R}$ (Real Numbers), and let f: $A \rightarrow B$, g: $B \rightarrow C$ be defined as follows:

$f(a) = a + 3$, and $g(b) = b^2 + 1$. Find:

(a) $(g \circ f)(-2)$

[3 marks]

(b) $(g \circ g)(x)$

[3 marks]

Question 10

Study the graph in Figure 1 and answer the questions that follow.

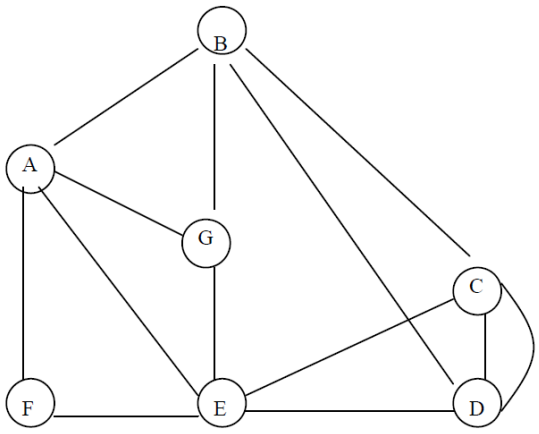


Figure 1

(a) Find the degree of each vertex in the graph.

[2 marks]

(b) Identify an Euler's cycle/path in the graph.

[3 marks]

(c) Identify a Hamilton's cycle/path in the graph.

[3 marks]

(d) Each edge in the graph was then given a weight as shown in Figure 2 below:

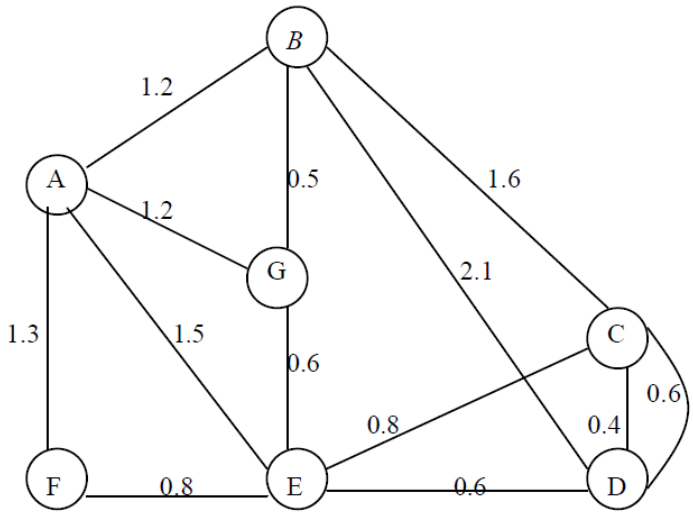


Figure 2

Find the Minimal Spanning Tree for the weighted graph using Prim's algorithm starting with vertex E.

[3 marks]

---End of Questions---