UNIVERSITI TENAGA NASIONAL

College of Information Technology

BACHELOR OF INFORMATION TECHNOLOGY (HONS.)

BACHELOR OF COMPUTER SCIENCE (HONS.)

FINAL EXAMINATION

SEMESTER I 2011/2012

DISCRETE STRUCTURES

(CSNB143)

August 2011

Time allowed: 3 hours + 10 minutes for reading

INSTRUCTIONS TO CANDIDATES

1. There is only **ONE (1) SECTION** to this paper: Section A.
2. Answer **ALL** questions in **Section A**.
3. The total marks for this exam is 100 marks.
4. Answer **ALL** the questions in the answer booklet provided.

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE INSTRUCTED TO DO SO

THIS QUESTION PAPER CONSISTS OF 8 PRINTED PAGES INCLUDING THIS PAGE

SECTION A – STRUCTURAL QUESTIONS (10 QUESTIONS, 100 MARKS)

Instruction: **This section contains TEN (10) questions. Answer ALL the questions. You need to show all the required working steps to show how you arrive at the solutions.**

Question 1

(a) Let the universal set U = {a, b, c, d, e, f, g, h, i}, A = {a, d, e, h}, B = {a, b, c, d, e} and set C = {b, d, f, h}. Illustrate each of the following with a Venn diagram, shade the areas stated in the questions and identify the elements of the shaded set.

i) $(A \cup B) - (C \cap B)$

ii) $C \cap (B - A)$

iii) $\overline{(A \oplus C)}$

[6 marks]

(b) If A = {3, 7},

i) what is |A|?

ii) what is | P(A) |?

[2 marks]

(c) Write a recursive formula for the sequence 2, 5, 7, 12, 19, 31.

[2 marks]

Question 2

(a) If $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$ find the value of *a*, *b*, *c* and *d.*

[2 marks]

(b) Given Boolean matrices as below:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find:

i) $A \vee B$

ii) $A \wedge B$

iii) $A \odot B$

[6 marks]

Question 3

Translate into symbols the following compound statements and give the form of the compound statement. In each case, list the statements *p, q, r*…

(a) Mathematics is easy and I do not like to study.

(b) If mathematics is easy and I like to study then I can get A.

(c) It is not both Mathematics is easy and I like to study.

(d) It is neither Mathematics is easy nor I like to study.

(e) $-1 \leq x \leq 2$

[10 marks]

Question 4

Using mathematical induction, prove the following:

(a) $n^3 + 2n$ is divisible by 3, given that $n \geq 3$

[5 marks]

(b) $a + ar + ar^2 + \ldots + ar^n = \frac{(ar^{n+1} - a)}{(r-1)}$, where $r \neq 1$ and n is a non-negative integer.

[5 marks]

Question 5

(a) A Negeri Sembilan (NS) netball team plays teams from other states in the competition so that they play four matches each month. Each match has three possible results for the NSW team – win, lose or draw. How many different sequences of results are possible in a month for NSW.

[2 marks]

(b) Find the number of ways to store 4 mathematics books, 3 history books, 3 chemistry books, 2 biology books in a shelf so that the same subject must be put next to each other.

[2 marks]

(c) How many choices are there if we want to make a 5 person committee from 12 persons, if the president has already been chosen?

[2 marks]

Question 6

(a) Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$. The relation R from A to B is defined by $R = \{(a,b) \in A \times B : a<b\}$.

i) Write down the elements of $A \times B$.

[2 marks]

ii) Write down the elements of the relation R. Is the relation R a function?

[2 marks]

(b) Given set $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 4), (2, 3), (3, 2), (4, 1), (4, 3)\}$.

i) Change the set of R into matrix, M_R and find the transitive closure using the Warshall algorithm.

[6 marks]

(c) Draw a directed graph with 5 vertices having the following in and out degrees stated in Figure 1:

	a	*b*	*c*	*d*	*e*
in-degree	1	1	1	1	0
out-degree	3	1	0	0	0

Figure 1

[3 marks]

Question 7

a) If $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4\}$. Identify if the following relation R from A to B is a function or not. Identify if it is also onto and/or one-to-one.

i) $R = \{(a, 1), (d, 2), (b, 2), (c, 1)\}$

ii) $R = \{(a, 1), (a, 2), (c, 1), (b, 2), (d, 2)\}$

[4 marks]

b) $f(a) = 3 + a^2$ and $g(b) = b(b + 2)$. Find:

i) $(g \circ f)(x)$ and $(g \circ f)(-2)$.

ii) $(f \circ g)(x)$ and $(f \circ g)(-3)$

[6 marks]

Question 8

a) Let $A = \{0, 1, 2, 3, 4, 5\}$. Write the following permutation on A in cycle notation.

$$P_A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 0 & 1 & 3 & 5 \end{pmatrix}$$

[2 marks]

b) Simplify the following composition of permutations:

$(1\ \ 2) \circ (2\ \ 3\ \ 1)$

[2 marks]

c) Write down the inverse permutation for (1 2 3 4).

[2 marks]

d) Write the matrix P_1 below as (dis)joint cycle, get its transposition product and identify either it is an even or odd permutation.

$$P_1 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 2 & 5 & 1 & 8 & 7 & 6 \end{bmatrix}$$

[4 marks]

Question 9

a) A building has 8 rooms labeled *A*, *B*... *H*. The room has doors that connect between rooms and the outside of the building, *I* as shown in Figure 2.

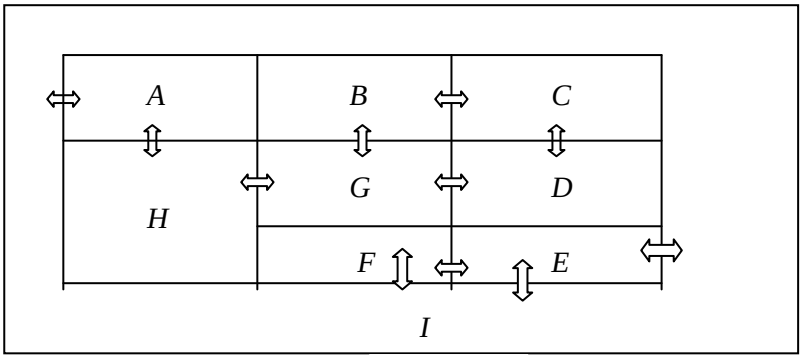

Figure 2

i) Transform the problem into graph.

[3 marks]

ii) Determine if the graph in (i) above has Euler cycle and/or path or Hamilton cycle and/or path.

[3 marks]

Question 10

a) Give the definition of a minimal spanning tree.

[2 marks]

b) Given a tree in Figure 3 below (with *f* designated as the root).

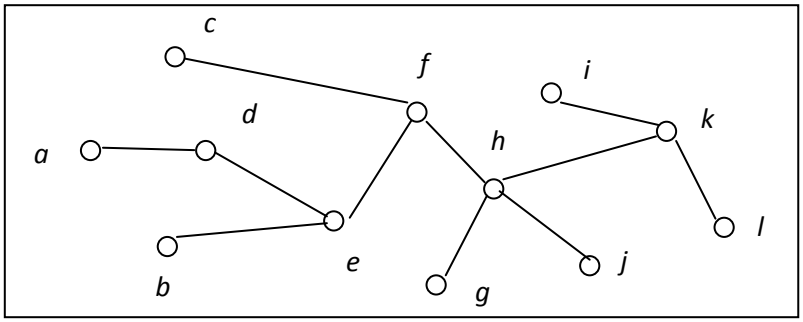

Figure 3

i) Redraw the tree to properly show the levels of each vertex.

ii) What is the height of the tree?

iii) What is the level of vertex *i* and *l*.

iv) List all vertices which are the descendants of *h*.

[8 marks]

c) The graph in Figure 4 is given a weight (in km) for each edge as illustrated in Figure 4.

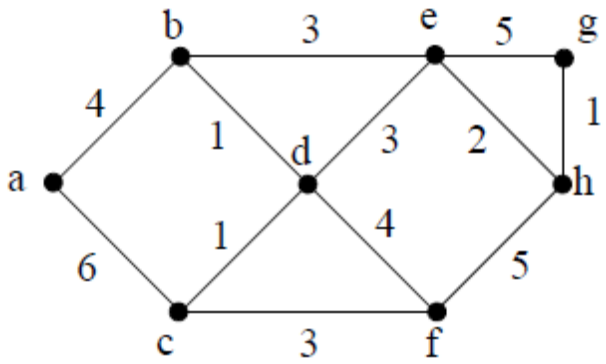

Figure 4

Find:

i) The Minimal Spanning Tree for the weightage graph from *a* using Prim approach.

[5 marks]

ii) The shortest distance.

[2 marks]

--- End of Questions ---