UNIVERSITI TENAGA NASIONAL

College of Information Technology

BACHELOR OF COMPUTER SCIENCE (HONS)
BACHELOR OF INFORMATION TECHNOLOGY (HONS)

FINAL EXAMINATION
SEMESTER II 2011/2012

DISCRETE STRUCTURES
(CSNB143)

December 2011

Time allowed: 3 hours + 10 minutes for reading

INSTRUCTIONS TO CANDIDATES.

1. The total marks for this exam is 100 marks.
2. There are **TWO (2) SECTIONS** to this paper: Section A and Section B
3. Answer **ALL** questions in the answer booklet provided.

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE INSTRUCTED TO DO SO.

THIS QUESTION PAPER CONSISTS OF 10 PRINTED PAGES INCLUDING THIS PAGE.

SECTION A: TRUE/FALSE (10 QUESTIONS, 10 MARKS)

Instruction: Indicate whether each of the following statements is True (T) or False (F)

1. A set should contain an ordered number of elements.

2. $A \cup B = B \cup A$ is one type of associative property.

3. A function f from A to B, denoted f: A $\rightarrow$ B is a relation from A to B such that for all $a \in$ Dom (f).

4. In matrix multiplication, (A + B)C = AC + BC.

5. 1, 2, 3, 4... is an example of a recursive and infinite sequence.

6. Connective OR is called conjunction for p and q.

7. In a relation R, the in-degree of R is the number of edges terminating on the vertex.

8. A path in a graph is called Euler path if it includes every edge exactly once.

9. Two statements p and q are said to be logically equivalent if p $\leftrightarrow$ q is a tautology.

10. Pigeonhole principle is a principle that ensures that data exists but there is no information to identify which data or what data.

SECTION B: SUBJECTIVES QUESTION (11 QUESTIONS, 90 MARKS)

Instruction: Answer all questions.

Question 1

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 4, 6, 8}, B={2, 4, 5, 9}, C = {x | x is a positive integer and $x^2 \leq 16$}, and D = {7, 8}. List down all elements of:

(a) C'

(b) A'

(c) $A \cup A'$

(d) $A \cap A'$

(e) $A \oplus B$

[5 marks]

Question 2

In a survey of 260 college students, the following data was obtained:

64 had taken a mathematics course,

94 had taken a computer science course,

58 had taken a business course,

28 had taken both a mathematics and a business course,

26 had taken both a mathematics and a computer science course,

22 had taken both computer science and business course, and

14 had taken all three types of courses.

Answer the following questions based on the above data. Show the complete working for your answers.

(a) How many students were surveyed who had taken none of these three types of courses?

[4 marks]

(b) Of the students surveyed, how many had taken only a computer science course?

[2 marks]

Question 3

Write a formula for the n^{th} term of the sequence below. Identify the formula as recursive or explicit.

(a) $S_1 = 1, 4, 7, 10, 13, 16...$

[2 marks]

(b) $S_2 = 0, 3, 8, 15, 24, 35....$

[2 marks]

Question 4

Given matrices A, B, C, D, E and F as below:

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 5 \\ 3 & 1 & 2 \end{pmatrix} \quad D = \begin{pmatrix} -3 & 2 \\ 4 & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 3 & 2 & -1 \\ 5 & 4 & -3 \\ 0 & 1 & 2 \end{pmatrix} \quad F = \begin{pmatrix} -2 & 3 \\ 4 & 5 \end{pmatrix}$$

Find:

(a) $A^T (D + F)$

[3 marks]

(b) $(BC)^T$

[2 marks]

(c) $FD + AB$

[2 marks]

(d) D^3

[3 marks]

Question 5

Given that p: Ali rich, q: Ali happy. Write the symbols for each of the statements below:

(a) Ali rich but happy.

[2 marks]

(b) Ali is poor if only Ali is happy.

[2 marks]

(c) If Ali rich, then Ali is not happy.

[2 marks]

(d) Ali either rich or happy.

[2 marks]

Question 6

Using mathematical induction, prove that for $\forall n \geq 0$,

$$3 + 3(5) + 3(5^2) + 3(5^3) + \ldots + 3(5^n) = \frac{3(5^{n+1} - 1)}{4}$$

[5 marks]

Question 7

Suppose an urn contains 15 balls, eight are red and seven are black. In how many ways can five balls be chosen so that:

(a) All five are red?

[2 marks]

(b) All five are black?

[2 marks]

(c) Two are red and three are black?

[2 marks]

(d) Three are red and two are black?

[2 marks]

(e) At least two are red?

[2 marks]

Question 8

Answers all questions based on the relation R diagraph in Figure 1 below.

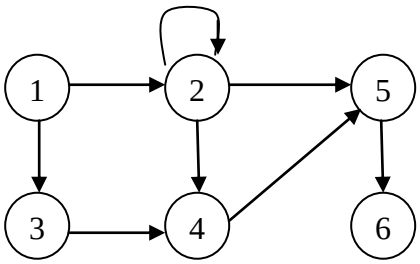

Figure 1

(a) Find R_2.

[5 marks]

(b) Draw the diagraph of R_2.

[5 marks]

Question 9

(a) Let A and B be the set of integer numbers. Suppose f amd g be functions such that *f*: A → B and *g*: B →A where $f(x) = x^2$ and $g(x) = \frac{(x+1)}{3}$. Find $f \circ g(x)$ and $g \circ f(x)$.

[4 marks]

(b) Let A = {1, 2, 3, 4, 5, 6}. Find:

(i) Permutation function of $(2, 3, 5, 4) \circ (1, 6, 3, 5)$.

[6 marks]

(ii) Product of transposition.

[2 marks]

(iii) For your answer in (ii), identify either it is odd or even permutation.

[1 mark]

Question 10

Consider the floor plan in Figure 2 below:

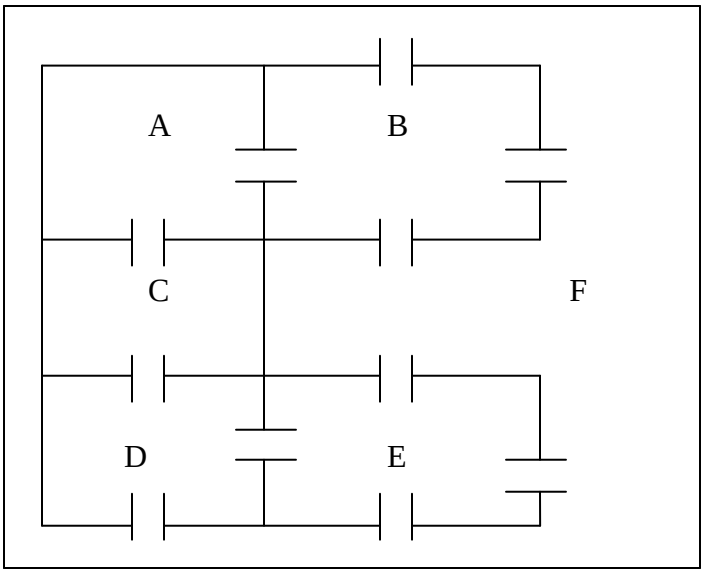

Figure 2

(a) Transfer it in a graph.

[5 marks]

(b) Find the degree of each vertex.

[3 marks]

(c) Determine if the graph above has an Euler circuit and/or path.

[2 marks]

(d) Determine if the graph above has a Hamilton circuit and/or path.

[2 marks]

Question 11

(a) Give the definition of minimal spanning tree.

[2 marks]

(b) The graph in Figure 3 is weighted (in km) for each edge. Find:

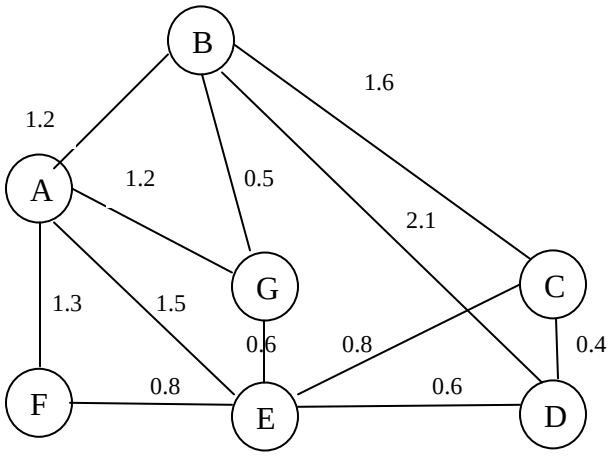

Figure 3

(i) The Minimal Spanning Tree for the weightage graph from E using Prim approach.

[3 marks]

(ii) The shortest distance.

[2 marks]