UNIVERSITI TENAGA NASIONAL

College of Information Technology

BACHELOR OF COMPUTER SCIENCE (HONS.)
BACHELOR OF INFORMATION TECHNOLOGY (HONS.)

FINAL EXAMINATION
SEMESTER I 2012/2013

DISCRETE STRUCTURES
(CSNB143)

September 2012

Time allowed: 3 hours + 10 minutes for reading

INSTRUCTIONS TO CANDIDATES

1. The total mark for this exam is 100.
2. There are **TWO (2)** sections namely Section A and B. Answer **ALL** the questions in the answer booklet provided.

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE INSTRUCTED TO DO SO

THIS QUESTION PAPER CONSISTS OF 8 PRINTED PAGES INCLUDING THIS PAGE

SECTION A: TRUE/FALSE (10 QUESTIONS, 10 MARKS)

<u>Instruction</u>: Indicate whether each of the following statement is TRUE (T) or FALSE (F).

1. Given a set $A = \{1, 2, 3, 4, 5\}$; another way to write the set A is $A = \{x \mid x$ is a positive integer and $x^2 \leq 25\}$.

2. A fourth value of $b_n = 2^n + 1$ is 16 where $n \geq 1$.

3. Two m x n matrices A and B, where $A = [a_{ij}]$ and $B = [b_{ij}]$, are said to be **equal** if $a_{ij} = b_{ij}$ for $1 \leq i \leq m$, $1 \leq j \leq n$; that is, if corresponding elements are the same.

4. Given a relation $R = \{(1, a), (2, b), (3, a), (3, c)\}$. Domain for R is Dom(R) = {1, 2, 3, 3} and Range for R is Ran(R) = $\{a, b, a, c\}$.

5. If p and q are statements, the compound statement if p then q, denoted by $p \leftrightarrow q$ is called a conditional statement or implication.

6. We say that relation R on set A is transitive if there exists a R b and b R c, there must exist a R c.

7. A relation R on set A is called Partial Order if R is reflexive, anti-symmetric and not transitive.

8. A function f from A to B, denoted f: $A \rightarrow B$ will contain one or more image under the element.

9. In a graph, any edge that connects from a vertex to the same vertex, such as a loop, contributes 2 to the degree of the vertex.

10. Vertices that share the same parent are called sibling and a vertex that has no children is called a leaf.

SECTION B: (10 QUESTIONS, 90 MARKS)
Instruction: Answer all the questions.

Question 1

In a survey of 350 students of College of IT, the following data was obtained:

72 had taken a Discrete Mathematic course

90 had taken a Computer Networking course

55 had taken a Software Engineering course

34 had taken both Discrete Mathematics and a Software Engineering course

23 had taken both a Discrete Mathematics and Computer Networking course

25 had taken both a Computer Networking and Software Engineering course

18 had taken all three types of courses.

(a) How many students were surveyed who had taken none of the three types of courses?

[4 marks]

(b) Of the students surveyed, how many had taken only a Computer Networking course?

[2 marks]

(c) Draw the Venn diagram for the survey result above.

[3 marks]

Question 2

Given the following matrices A and B:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Find:

(a) $A \vee B$

(b) $A \wedge B$

(c) $A \bullet B$

[6 marks]

Question 3

(a) Prove that $(P \rightarrow Q)$ and $(\sim Q \rightarrow \sim P)$ are logically equivalent.

[5 marks]

(b) Let the domain be *"all students who study in UNITEN"* and $P(x)$: *"x is a student takes CSNB143 subject"* be a proposition. What is the truth value of the following statements? Justify your answer

i. $\forall x\, P(x)$

ii. $\exists x\, P(x)$

[5 marks]

Question 4

Using the Induction Principle of Mathematics, show that:

$n^3 - n$ is always divisible by 3, for integers $n \geq 2$.

[6 marks]

Question 5

(a) Sam is buying fruit trees for his garden. He plans to plant one peach, one apple, one plum and one cherry tree. The nursery recommends two varieties of peach, four of apple, six of plum and three of cherry for his area. How many possible different groups of trees can he plant? If Sam has already decided on the cherry, how many choices are left for the other trees?

[4 marks]

(b) How many distinct sets of 3 differently colored scarves can be bought if the shop has scarves in 8 different colors?

[4 marks]

Question 6

(a) Let $S = \{1, 2, 3, 4\}$ and a pair of $(x, y) \in R$ where R is defined as $x \leq y$.

(i) List all members of R.

[2 marks]

(ii) Represent R as a diagraph.

[2 marks]

(iii)Find the in-degree and out-degree for each vertex.

[2 marks]

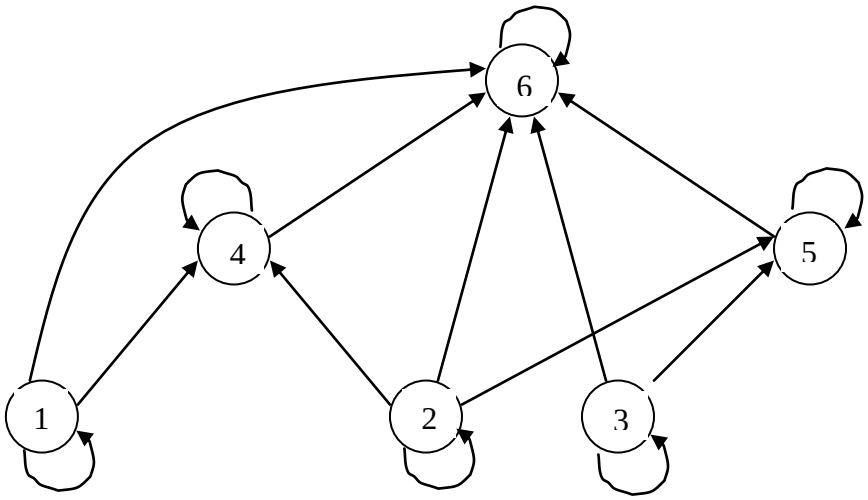
(b) Consider $M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Find the matrix of the transitive closure using WARSHALL algorithm.

[5 marks]

Question 7

A diagraph is as given below.



Find:

(a) The Hasse Diagram.

[5 marks]

(b) **FIVE (5)** valid topological sorting orders.

[5 marks]

Question 8

(a) Let $A, B, C \in$ Real Numbers, and let f: $A \rightarrow B$, g: $B \rightarrow C$ is explained as $f(a) = 3 + a^2$ and $g(b) = b(b + 2)$. Find;

(i) $(g \circ f)(-2)$

[1 marks]

(ii) $(f \circ g)(x)$

[1 marks]

(b) Given set $A = \{1, 2, 3, 4, 5, 6\}$.

(i) Find the function permutation of (2, 5, 6, 1). Label it as *P1*.

[2 marks]

(ii) Find the function permutation of (6, 3, 2, 4). Label it as *P2*.

[2 marks]

(iii) Using (**b (i)**) above, find $P1^{-1}$.

[2 marks]

(iv) Using (**b (i)**) and (**b (ii)**) find $P2 \circ P1$.

[2 marks]

(v) Find the product of disjoint cycles of the answer of (**b (iv)**).

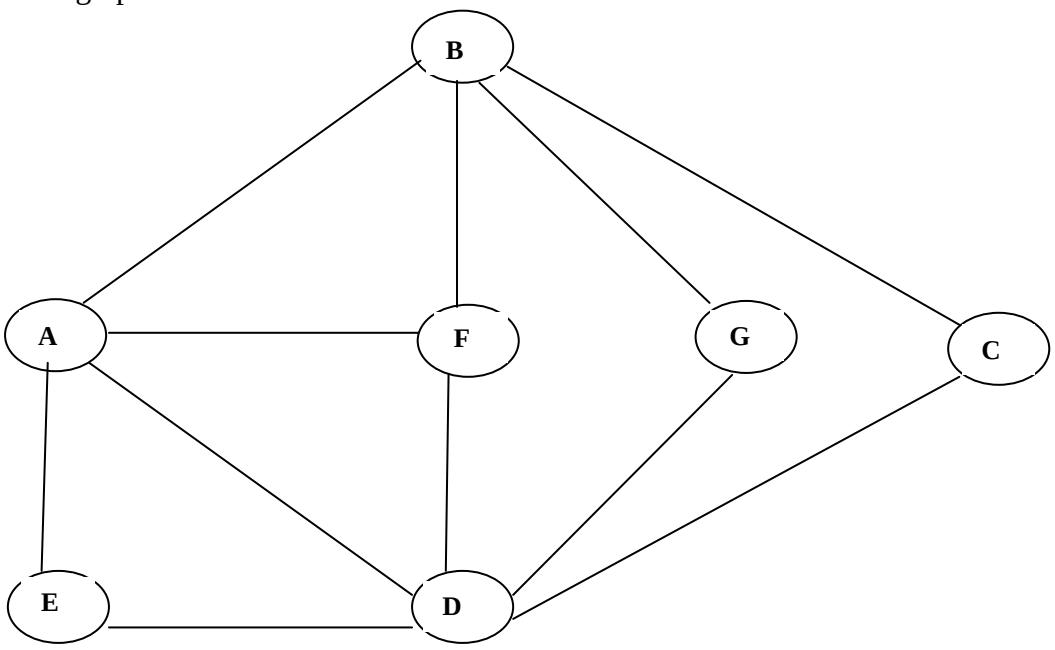
[2 marks]

(vi) Find the transposition product of (**b (v)**). Identify whether it is an even or odd permutation.

[2 marks]

Question 9

Given a graph as follows:



(a) Find the degrees of vertices *A, B, C* and *D*.

[2 mark]

(b) Define and describe how to determine an Euler's path. Identify it in the graph above.

[3 marks]

(c) Define and describe how to determine a Hamilton's cycle/path. Identify it in the graph above.

[3 marks]

Question 10

Consider the following table

	A	B	C	D	E	F	G	H	I
A	-	*4*	-	*5*	-	-	*2*	-	-
B	*4*	-	-	-	-	*3*	*1*	*5*	-
C	-	-	-	*2*	*1*	*3*	-	-	*3*
D	*5*	-	*2*	-	-	*2*	*1*	-	-
E	-	-	*1*	-	-	-	-	*1*	*2*
F	-	*3*	*3*	*2*	-	-	-	*2*	-
G	*2*	*1*	-	*1*	-	-	-	-	-
H	-	*5*	-	-	*1*	*2*	-	-	*1*
I	-	-	*3*	-	*2*	-	-	*1*	-

(a) Draw the minimal spanning tree.

[6 marks]

(b) Find the minimum distance.

[2 marks]

---------- End of Questions ----------