

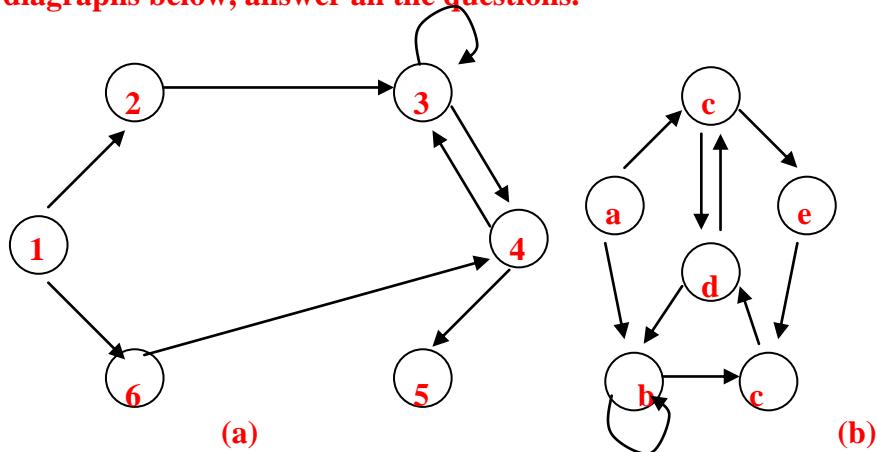
TUTORIAL8: RELATIONS

1. Change the matrices on set A below into set and digraph. Then read the in-degree and out-degree.

a) $A = \{1, 2, 3, 4\}$ and $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

b) $A = \{a, b, c, d, e\}$ and $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

2. From the digraphs below, answer all the questions.



- Change diagram (a) into set.
 - Change diagram (b) into matrix.
 - Draw the digraph of R^2 for (a).
 - Find M_R^2 for (b).
3. Consider relations on set $A = \{1, 2, 3, 4\}$. Determine either the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric and/or transitive?

$$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$$

$$R_2 = \{(1, 1), (1, 2), (2, 1)\}$$

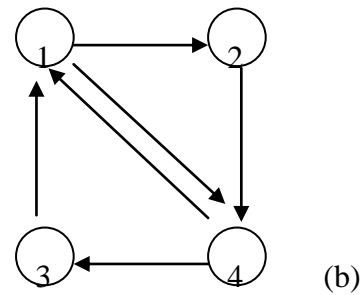
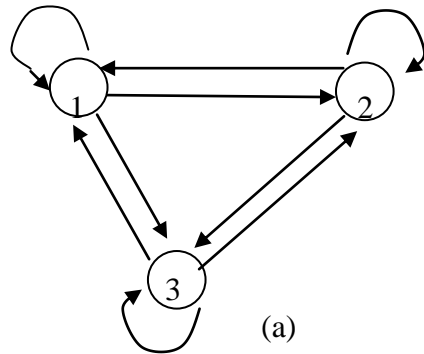
$$\mathbf{R}_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$$

$$\mathbf{R}_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$$

$$\mathbf{R}_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$$

$$\mathbf{R}_6 = \{(3, 4)\}$$

4. From the diagrams below, determine either the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric and/or transitive?



5. From the matrices below, determine either the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric and/or transitive?.

a)
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

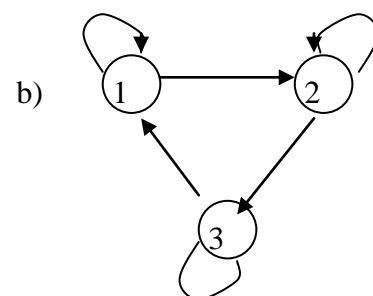
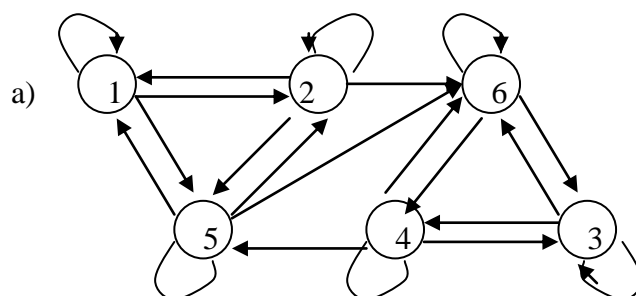
b)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. Given $A = \{a, b, c\}$. Determine either R is an equivalence relation or not.

a)
$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

b)
$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7. In diagrams below, determine either R is an equivalence relation or not.



8. In relations on sets $A = B = \{1, 2, 3\}$ below, find:

- a) $R \cap S$ b) $R \cup S$ c) S^{-1}
 d) R^{-1} e) \overline{R} f) \overline{S}
 g) $S \circ R$ h) $R \circ S$ i) $R \circ R$

i) $R = \{(1, 1), (1, 2), (2, 3), (3, 1)\}; S = \{(1, 2), (2, 1), (3, 1), (3, 2), (3, 3)\}.$

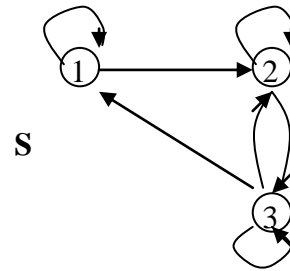
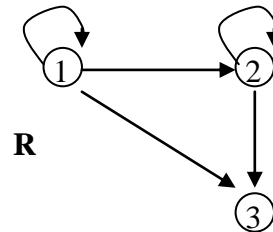
ii) $R = \{(1, 1), (2, 1), (3, 2), (3, 3)\}; S = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

iii) $A = B = \{1, 2, 3, 4\}$

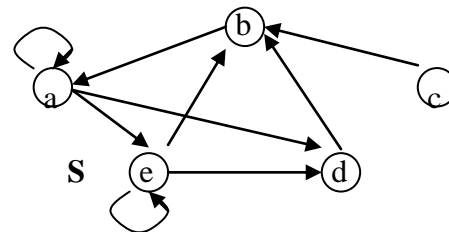
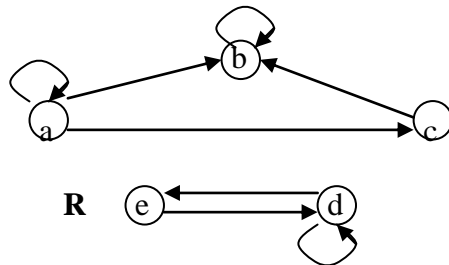
$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_S = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

iv)



v) $A = B = \{a, b, c, d, e\}$



9. Let $A = \{1, 2, 3, 4\}$. R is given by matrices R and S below. Find the transitive closure by using Warshall Algorithm.

a) $M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

b) $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$