

Digital Logic Design (CSNB163)

Module 1

Number Systems

- Number system consists of an ordered set of symbols called digit.
- The **radix (r)** or **base** is the total number of digits allowed in the system.
- Common number systems includes
 - **decimal (r = 10)** {0,1,2,3,4,5,6,7,8,9}
 - **binary (r = 2)** {0,1}
 - **octal (r = 8)** {0,1,2,3,4,5,6,7}
 - **hexadecimal (r = 16)** {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}

Number Representations

- A number may appear in 2 parts:

- **integer part**



123.45

- **fractional part**

which are separated by radix point (.).

- Numbers can be represented in 2 notations:

- **Position notation**

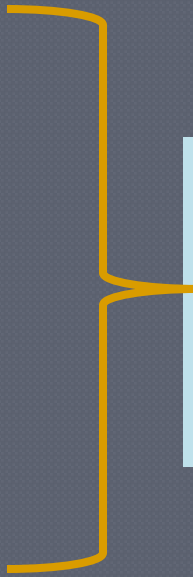
123.45

- **Polynomial notation**

$$1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2}$$

Arithmetic Operation

- Arithmetic operations can be performed on numbers regardless of their radices.
- Common arithmetic operation includes:
 - Addition (+)
 - Subtraction (-)
 - Multiplication (\times)
 - Division (\div)



Decimal
Binary
Octal
Hexadecimal

Arithmetic Operation (Addition)

◉ Decimal

$$\begin{array}{r} 1234_{10} \\ + 4567_{10} \\ \hline 5801_{10} \end{array}$$

◉ Binary

$$\begin{array}{r} 10011010010_2 \\ + 1000111010111_2 \\ \hline 1011010101001_2 \end{array}$$

◉ Octal

$$\begin{array}{r} 2322_8 \\ + 10727_8 \\ \hline 13251_8 \end{array}$$

◉ Hexadecimal

$$\begin{array}{r} 4D2_{16} \\ + 11D7_{16} \\ \hline 16A9_{16} \end{array}$$

Arithmetic Operation (Subtraction)

○ Decimal

$$\begin{array}{r} 4567_{10} \\ - 1234_{10} \\ \hline 3333_{10} \end{array}$$

○ Binary

$$\begin{array}{r} 1000111010111_2 \\ - 10011010010_2 \\ \hline 110100000101_2 \end{array}$$

○ Octal

$$\begin{array}{r} 10727_8 \\ - 2322_8 \\ \hline 6405_8 \end{array}$$

○ Hexadecimal

$$\begin{array}{r} 11D7_{16} \\ - 4D2_{16} \\ \hline D05_{16} \end{array}$$

Arithmetic Operation (Multiplication)

○ Decimal

$$\begin{array}{r} 4567_{10} \\ \times \quad 2_{10} \\ \hline 9134_{10} \end{array}$$

○ Octal

$$\begin{array}{r} 10727_8 \\ \times \quad 2_8 \\ \hline 21656_8 \end{array}$$

○ Binary

$$\begin{array}{r} 1000111010111_2 \\ \times \quad 10_2 \\ \hline 10001110101110_2 \end{array}$$

○ Hexadecimal

$$\begin{array}{r} 11D7_{16} \\ \times \quad 2_{16} \\ \hline 23AE_{16} \end{array}$$

Arithmetic Operation (Division)

◉ Decimal

$$\begin{array}{r} 1234_{10} \\ \div \quad \underline{2}_{10} \\ \underline{617}_{10} \end{array}$$

◉ Octal

$$\begin{array}{r} 2322_8 \\ \div \quad \underline{2}_8 \\ \underline{1151}_8 \end{array}$$

◉ Binary

$$\begin{array}{r} 10011010010_2 \\ \div \quad \underline{\hspace{1.5cm}10}_2 \\ \underline{1001101001}_2 \end{array}$$

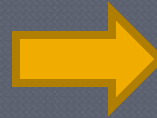
◉ Hexadecimal

$$\begin{array}{r} 4D2_{16} \\ \div \quad \underline{2}_{16} \\ \underline{269}_{16} \end{array}$$

Number Base Conversion

- To convert a number in one base to another.
- May involve conversion of fractional parts.

Decimal
Binary
Octal
Hexadecimal



Decimal
Binary
Octal
Hexadecimal

- There are many conversion techniques, however we shall concentrate on:
 - **Radix based conversion**
 - **Grouping based conversion**

Radix Based Conversion

○ Recaps: A number may appear in 2 parts:

- **integer part**



123.45

The diagram shows the number 123.45 in red text inside a light blue box. Two yellow dashed circles are drawn around the '123' and '.45' respectively. A yellow arrow points from the 'integer part' label to the '123' circle, and another yellow arrow points from the 'fractional part' label to the '.45' circle.

- **fractional part**

which are separated by radix point (.).

○ For radix based conversion:

- integer part – **divide by radix**

- fractional part – **multiply by radix**

Radix Based Conversion (Example1)

- Convert 1234 decimal into binary

Radix 2 Divide by radix 2	2	1234	
	2	617	0
	2	308	1
	2	154	0
	2	77	0
	2	38	1
	2	19	0
	2	9	1
	2	4	1
	2	2	0
		1	0

Answer

 10011010010_2

Radix Based Conversion (Example2)

- Convert 1234 decimal into octal

Radix 8

Divide
by radix 8

8	1234	
8	154	2
8	19	2
	2	3

Answer

2322₈

Radix Based Conversion (Example3)

- Convert 1234 decimal into hexadecimal

Radix 16

Divide
by radix 16

16	1234	
16	77	2
	4	D

Answer

4D2₁₆

Radix Based Conversion (Example4)

- Convert 0.6875 decimal into binary

Radix 2

Multiply by radix 2

Fraction	Radix	Total (Fraction x Radix)	Integer	Fraction
0.6875	2	1.375	1	0.375
0.375	2	0.75	0	0.75
0.75	2	1.5	1	0.5
0.5	2	1	1	0

Answer

0.1011₂

Radix Based Conversion (Example5)

- Convert 0.513_{10} to base 16 (up to 4 fractional point)

Radix 16

Multiply by radix 16

Fraction	Radix	Total (Fraction x Radix)	Integer	Fraction
0.513	16	8.208	8	0.208
0.208	16	3.328	3	0.328
0.328	16	5.248	5	0.248
0.248	16	3.968	3	0.968

Answer

0.8353_{16}

Grouping Based Conversion

○ Recaps: A number may appear in 2 parts:

- **integer part**



123.45

- **fractional part**

which are separated by radix point (.).

○ For radix based conversion:

- integer part – **divide by radix**

- fractional part – **multiply by radix**

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End of Module 1