Digital Logic Design (CSNB163)

Module 4

Boolean Algebra-Digital Logic Design

- Boolean Algebra is used to **simplify** the design of digital logic circuits.

Complicated design with many digital logic circuits

Boolean Algebra

Simpler design with less digital logic circuits

Both performs similar function but the latter is cheaper

- The design simplification are based on:
  - **Postulates of Boolean Algebra**
  - **Basic Property of Boolean Algebra**
  - **Basic Theorems of Boolean Algebra**

Two Valued Boolean Algebra

- A two valued Boolean Algebra is defined on a **set (*B*)** of two elements:

$$B = \{0,1\}$$

- A two valued Boolean Algebra reflects the basis for digital logic circuit (i.e. whereby digital signals being the IO to the digital logic circuit can only be either 0 or 1).
- A two valued Boolean Algebra satisfies:
  - **Postulates of Boolean Algebra**
  - **Basic Property of Boolean Algebra**
  - **Basic Theorems of Boolean Algebra**

Postulates of Boolean Algebra

- **Postulate 1: Closure**

  The Boolean system is closed with respect to binary operator + and . since for every possible combination of Boolean values from set {1, 0} , it produces a Boolean result from set {1, 0}.

Postulates of Boolean Algebra

- **Postulate 2: Identity Element**
- The element 0 is an identity element with respect to + operator:

$$a + 0 = a$$

- The element 1 is an identity element with respect to . operator:

$$a \cdot 1 = a$$

Postulates of Boolean Algebra

- **Postulate 3: Commutative**

  Commutativity of the + operation

  $$a + b = b + a$$

  Commutativity of the . operation

  $$a \,.\, b = b \,.\, a$$

Postulates of Boolean Algebra

- **Postulate 4: Associative**

Associativity of the + operation

$$a + (b + c) = (a + b) + c$$

Associativity of the . operation

$$a . (b . c) = (a . b) . c$$

Postulates of Boolean Algebra

- **Postulate 5: Distributive**

Distributivity of the + operation

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$

Distributivity of the . operation

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

Postulates of Boolean Algebra

- **Postulate 6: Complement**

For every *a* in K set, there exists a unique element called *a'* (complement of *a*) such that

$$a + a' = 1$$

and

$$a \,.\, a' = 0$$

Basic Property of Boolean Algebra

- **Duality**

  If an expression is valid in Boolean Algebra, the dual of the expression is also valid.

  The dual expression is done by:

  - **replacing** all **+** operators with **.** and vice versa
  - **replacing** all **1**s with **0**s and vice versa

  Example : $a + (b . c) = (a + b) . (a + c)$
  Thus thorough duality : $a . (b+c) = (a . b) + (a . c)$

Basic Theorems of Boolean Algebra

- **Theorem 1: Idempotency**

  The output value does not change by its input multiplication whereby:

  $$x + x = x$$

  and based on duality property:

  $$x \, . \, x = x$$

Basic Theorems of Boolean Algebra

- **Theorem 2: Null Element**

  The output value is not affected by the changes in the input value, whereby:

  $$x + 1 = 1$$

  and based on duality property:

  $$x \cdot 0 = 0$$

Basic Theorems of Boolean Algebra

- **Theorem 3: Involution**

  The double inverse output value of an input is equivalent to the input:

$$(x')' = x$$

Basic Theorems of Boolean Algebra

- **Theorem 4: Redundancy**

This theorem is the result of the application of several other theorems that eliminates redundancy, whereby:

$$a + (a' \cdot b) = a + b$$

and based on duality property:

$$a \cdot (a' + b) = a \cdot b$$

Basic Theorems of Boolean Algebra

- **Theorem 4: Redundancy**

  Proving:

  $a + (a' \cdot b)$
  $= (a + a')(a + b)$ (via Postulate 5: Distributive)
  $= (1)(a+b)$ (via Postulate 6: Complement)
  $= a + b$ (via Postulate 2: Identity Element)

a	b	a + (a' . b)	a + b
0	0	0 + (1 . 0) = 0	0 + 0 = 0
0	1	0 + (1 . 1) = 1	0 + 1 = 1
1	0	1 + (0 . 0) = 1	1 + 0 = 1
1	1	1 + (0 . 1) = 1	1 + 1 = 1

Basic Theorems of Boolean Algebra

- **Theorem 5: DeMorgan's Law**

This theorem is based on DeMorgan's Law, whereby:

$$\overline{(a + b)} = \overline{a} \;.\; \overline{b}$$

and based on duality property:

$$\overline{(a \;.\; b)} = \overline{a} + \overline{b}$$

Basic Theorems of Boolean Algebra

- **Theorem 6: Absorption**

This theorem is the result of the application of several other theorems that neglects a certain input variable, whereby:

$$a + (a \cdot b) = a$$

and based on duality property:

$$a \cdot (a + b) = a$$

Basic Theorems of Boolean Algebra

- **Theorem 6: Absorption**

  Proving:

a + (a . b)	
= (a . 1) + (a . b)	(via Postulate 2: Identity Element)
= (a)(1+b)	(via Postulate 5: Distributive)
= a (1)	(via Theorem 2: Null Element)
= a	(via Postulate 2: Identity Element)

a	b	a + (a . b)
0	0	0 + (0 . 0) = 0
0	1	0 + (0 . 1) = 0
1	0	1 + (1 . 0) = 1
1	1	1 + (1 . 1) = 1

Logic Design Simplification via Boolean Algebra

- As mentioned in the introduction, we can use Boolean Algebra to **simplify** the design of digital logic circuit. This is made possible via basic postulates, property and theorems of Boolean Algebra itself.
- By simplifying the digital logic design, fewer gates (and wiring) are used to achieve the same realization, thus **more cost effective**.
- However, since Boolean Algebra can be simplified in several different ways, there is **no standard rule** to guarantee the final answer.

Logic Design Simplification (Example 1)

- Simplify the following Boolean function

$$F1 = x(x' + y)$$

using basic postulates, property and theorems of Boolean Algebra.

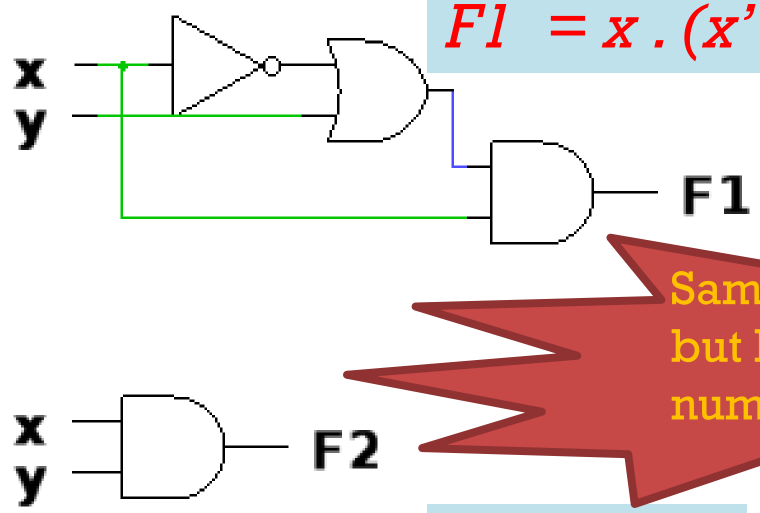
$x(x' + y)$	
$= (x \,.\, x') + (x \,.\, y)$	(via Postulate 5: Distributive)
$= (0) + (x \,.\, y)$	(via Postulate 6: Complement)
$= x.y$	(via Postulate 2: Identity Element)

Logic Design Simplification (Example1 – Truth Table)

- The truth diagram:

x	y	F1 = x (x' + y)	F2 = x . y
0	0	0 . (1 + 0) = 0	0 . 0 = 0
0	1	0 . (1 + 1) = 0	0 . 1 = 0
1	0	1 . (0 + 0) = 0	1 . 0 = 0
1	1	1 . (0 + 1) = 1	1 . 1 = 1

Logic Design Simplification (Example1 – Circuit Diagram)



$F1 = x \cdot (x' + y)$

$F2 = x \cdot y$

Same realization of F1 and F2, but F2 is simpler with less num. of gates & wiring!!

Logic Design Simplification (Example2)

- Simplify the following Boolean function

$$F1 = x'y'z + x'yz + xy'$$

using basic postulates, property and theorems of Boolean Algebra.

$x'y'z + x'yz + xy'$

$= (x'z)(y' + y) + (xy')$ (via Postulate 5: Distributive)

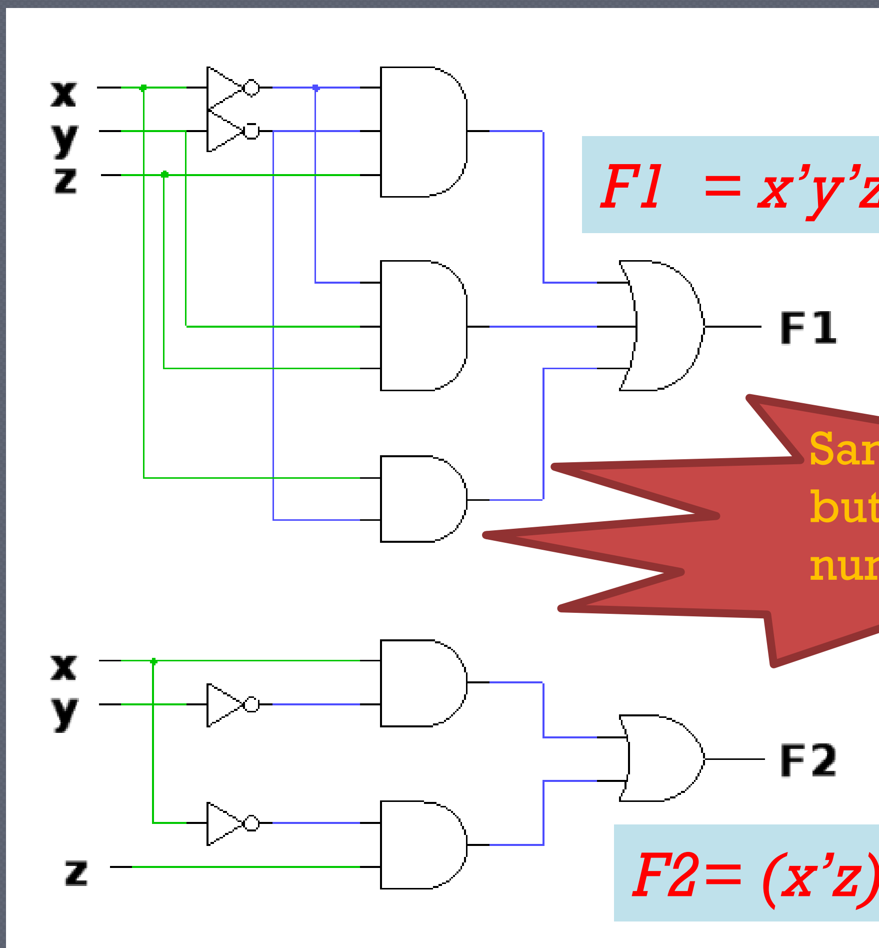
$= (x'z) + (xy') = F2$ (via Postulate 6: Complement)

Logic Design Simplification (Example2 – Truth Table)

- The truth diagram:

x	y	z	F1 = x'y'z + x'yz + xy'	F2 = xy' + x'z
0	0	0	1.1.0 + 1.0.0 + 0.1 = 0	0.1 + 1.0 = 0
0	0	1	1.1.1 + 1.0.1 + 0.1 = 1	0.1 + 1.1 = 1
0	1	0	1.0.0 + 1.1.0 + 0.0 = 0	0.0 + 1.0 = 0
0	1	1	1.0.1 + 1.1.1 + 0.0 = 1	0.0 + 1.1 = 1
1	0	0	0.1.0 + 0.0.0 + 1.1 = 1	1.1 + 0.0 = 1
1	0	1	0.1.1 + 0.0.1 + 1.1 = 1	1.1 + 0.1 = 1
1	1	0	0.0.0 + 0.1.0 + 1.0 = 0	1.0 + 0.0 = 0
1	1	1	0.0.1 + 0.1.1 + 1.0 = 0	1.0 + 0.1 = 0

Logic Design Simplification (Example2 – Circuit Diagram)



$F1 = x'y'z + x'yz + xy'$

Same realization of F1 and F2, but F2 is simpler with less num. of gates & wiring!!

$F2 = (x'z) + (xy')$

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End of Module 4