Digital Logic Design (CSNB163)

Module 6

Digital Logic Design Simplification

- In Module 4, we have learned how to simplify the design of digital logic circuits by manipulating **Boolean Algebra**. However, this method lacks of standardization.
- There is another alternative of design simplification which is through the use of Karnaugh Map or K-Map.

Complicated design with many digital logic circuits

K-Map Simplification

Simpler design with less digital logic circuits

This method is simpler and more standardized!

Karnaugh Map

- **Karnaugh Map (or K-Map)** is a **2D** pictorial form of a truth table that describes the relationship between input and output variables.
- K-map simplification can be carried out through:
  - **Sum of Product simplification**
  - **Product of Sum simplification**
- In this module, we shall only look at the Sum of Product simplification.

SOP - Karnaugh Map Simplification

- In Sum of Product - Karnaugh Map simplification, the K-Map is made up of cells representing all possible combination of input variables in **minterms** format. Each cell is denoted as:
  - **1** for minterm that **corresponds** to the Boolean equation
  - **0** for minterm that **does NOT correspond** to the Boolean equation
- Simplification is done through **grouping** of all cell with **value ‘1’**

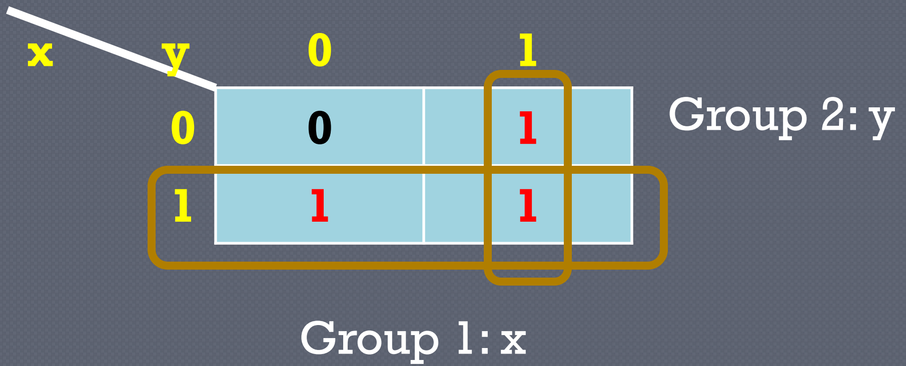
Karnaugh Map Simplification Tips

- Always try to group cells in **power of 2** (e.g. 2^0, 2^1, 2^2..etc).
- Group **as many cells** as possible. The larger the group, the fewer number of input variables.
- Make sure that **all** minterms with value '1' is covered!!!
- Sometimes, there may be **more than 1** simplification result.

Karnaugh Map (2 input variables)

- If we have 2 input variables:
  - possible number of combinations $= 2^2 = 4$
  - i.e. 4 possible minterms, thus a K-Map with 4 squares
- E.g.

$F(x,y) = x'y + xy' + xy$



Thus simplified $F(x,y) = x + y$

Karnaugh Map (3 input variables)

- If we have 3 input variables:
  - possible number of combinations $= 2^3 = 8$
  - i.e. 8 possible minterms, thus a K-Map with 8 squares
- E.g.

F(x, y, z) = x'yz' + x'yz + xy'z' + xy'z

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

Group 2: x'y

Group 1: xy'

Thus simplified F(x, y, z)= xy' + x'y

Karnaugh Map (4 input variables)

- If we have 4 input variables:
  - possible number of combinations $= 2^4 = 16$
  - i.e. 16 possible minterms, thus a K-Map with 16 squares
- E.g.

F(a, b, c, d)= a'b'c'd + a'bc'd + abc'd + abcd

Group 1: a'c'd

ab \ cd	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	0	1	1	0
10	0	0	0	0

Group 2: abd

Thus simplified F(a, b, c, d)= a'c'd + abd

Exercise 1

- Simplify the following equation using Karnaugh map:

$$F(x, y, z) = x'y'z' + x'yz' + xy'z' + xy'z$$

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	0	0

Group 2: $x'z'$

Group 1: xy'

Thus simplified $F(x, y, z) = xy' + x'z'$

Exercise 2

- Simplify the following equation using Karnaugh map:

$$F(x, y, z) = xy'z' + xy'z + x'yz + xy$$

x \ yz	00	01	11	10
0	0	0	1	0
1	1	1	1	1

Group 2: yz

Group 1: x

Thus simplified $F(x, y, z) = x + yz$

Exercise 3

- Simplify the following equation using Karnaugh map:

F(a, b, c, d)= a’b’cd + a’b’cd’ + ab’c’d’ + ab’cd’

ab \ cd	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

Group 1: a’b’c

Group 2: ab’d’

Thus simplified F(a, b, c, d)= a’b’c + ab’d’

Exercise 4

- Simplify the following equation using Karnaugh map:

F(a, b, c, d)= a’b’c’ + bc’d + ab’c’ + a’b’cd’ + ab’cd’

Group 1: c’d

ab \ cd	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	1	0	0
10	1	1	0	1

Group 2: b’d’

Thus simplified F(a, b, c, d)= cd’ + b’d’

Don’t Care Terms

- In some applications, the function is **not specified** for certain combinations of the variables – this combination is known as a **‘don’t care’ term**.
- A ‘don’t care’ term maybe assumed to be **either 0 or 1.**
- An **X** is used to indicate the ‘don’t care’ function.
- The X can be replaced by **either** 1 or 0, depending on which combination that gives the **simplest expression**.

Exercise 5

- By using K-map, simplify the following equation: $F(x, y, z) = \sum(5,6,7)$
  whereby the 'don't care' terms: $d(x, y, z) = \sum(3,4)$

xyz	Designation
000	m0
001	m1
010	m2
011	m3
100	m4
101	m5
110	m6
111	m7

x \ yz	00	01	11	10
0	0	0	X	0
1	X	1	1	1

Group 1 = x

$F(x, y, z) = x$

Exercise 6

- By using K-map, simplify the following equation: $F(a, b, c, d) = \sum(1,3,7,11,15)$
  whereby the 'don't care' terms:
  $d(a, b, c, d) = \sum(0,2,5)$

abcd	Designation	abcd	Designation
0000	m0	1000	m8
0001	m1	1001	m9
0010	m2	1010	m10
0011	m3	1011	m11
0100	m4	1100	m12
0101	m5	1101	m13
0110	m6	1110	m14
0111	m7	1111	m15

ab \ cd	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

Exercise 6 (Solution 1)

- Solution 1

ab \ cd	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

Group 1 : a’b’

Group 2 : cd

F(a, b,c, d)= a’b’ + cd

Exercise 6 (Solution 2)

- Solution 2

Group 1 : a'd

ab \ cd	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

Group 2 : cd

F(a, b,c, d)= a'd + cd

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End of Module 6