The Discrete-time Unit Impulse Sequence

\[ \delta[n] \]
Impulse Function

volt

time
Delayed Unit Impulse
The Unit Step Sequence

\[ u[n] = 0, \quad n < 0, \]

\[ u[n] = 1, \quad n \geq 0. \]
Relationship between Unit Impulse & Unit Step Sequences

Discrete-time unit impulse is the first difference of the discrete-time unit step.

\[ \delta[n] = u[n] - u[n-1]. \]
Relationship between Unit Impulse & Unit Step Sequences

Discrete-time unit step is the running sum of the discrete-time unit impulse or unit sample.

\[ u[n] = \sum_{m=-\infty}^{n} \delta[m]. \]
Interval of summation

\[ \delta [m] \]

Interval of summation
Alternative way of expressing the unit step as the summation (superposition) of delayed unit impulse

Changing the variable: _

let \( k = n - m \),
\[ \therefore m = n - k. \]

\[ u[n] = \sum_{k=\infty}^{0} \delta[n - k], \]

or \( u[n] = \sum_{k=0}^{\infty} \delta[n - k] \).
\[ \delta[n - k] \]

\[ u[n] = \sum_{k=0}^{\infty} \delta[n - k]. \]

Interval of summation

\[ n < 0 \]

\[ 0 \]

\[ k \]

\[ n \geq 0 \]
\[ u[n] = \sum_{k=0}^{\infty} \delta[n-k]. \]
The Sampling Property of the Unit Impulse.

The unit impulse sequence can be used to sample the value of a signal at \( n = 0 \). Since \( \delta[n] \) is nonzero \((=1)\) only for \( n = 0 \),

\[
\therefore x[n] \delta[n] = x[0] \delta[n].
\]

It follows that generally since \( \delta[n - n_o] = 1 \) for \( n = n_o \), then

\[
x[n] \delta[n - n_o] = x[n_o] \delta[n - n_o].
\]
\[ n \delta \quad 0 \quad x[0] \quad x[-1] \quad x[n_0] \]

\[ \delta [n] \]

\[ \delta [n - n_0] \]
The Continuous-time Unit Step

Definition of unit step function: –
\[ u(t) = 0, \quad t < 0, \]
\[ u(t) = 1, \quad t > 0. \]

This function is discontinuous at \( t = 0 \).

![Diagram of the unit step function](image-url)
The Continuous-time Unit Impulse Function.

Definition:

\[ \delta(t) = 0, \quad t \neq 0, \]

\[ \delta(t) = 1 \text{(area)}, \quad t = 0, \]
Relationship between continuous-time unit step and unit impulse.

\[
u(t) = \int_{-\infty}^{t} \delta(\tau) \, d\tau.
\]

\[
\delta(t) = \frac{du(t)}{dt}.
\]
Integration of Unit Impulse to get Unit Stup

\[ \int_{-\infty}^{0} \delta (\tau) \, d\tau = 0 \]

Interval of integration

\[ \int_{0}^{\infty} \delta (\tau) \, d\tau = 1 \]
Discontinuity at $t=0$, poses problem of differentiation.

\[
\delta_\nabla(t) = \lim_{\nabla \to 0} \frac{d u_\nabla(t)}{dt}, \quad \delta(t) = \lim_{\nabla \to 0} \delta_\nabla(t)
\]
Unit and scaled Impulse

\[ \delta(t) \]

\[ k \delta(t) \]
Scaled impulse and relationship of unit step & impulse.

\[ \int_{-\infty}^{t} k \delta (\tau) d\tau = ku(t). \]

Letting \( \sigma = t - \tau \), and scale \( k = 1 \),

\[ u(t) = \int_{-\infty}^{t} \delta (\tau) d\tau = \int_{0}^{0} \delta (t - \sigma)(-d\sigma), \]

or equivalently:

\[ u(t) = \int_{0}^{\infty} \delta (t - \sigma) d\sigma. \]
Relationship of Unit and Impulse

\[ \delta(t-\sigma) \]

Interval of integration

\[ t < 0 \]

\[ 0 \]

\[ t > 0 \]

\[ \sigma \]
Consider $x_1(t) = x(t)\delta_\nabla(t)$. 
For $\nabla$ sufficiently small, $x(t)$ is constant over interval $\nabla$, 
$\therefore x(t)\delta_{\nabla}(t) \approx x(0)\delta_{\nabla}(t)$.

Since $\delta(t) = \delta_{\nabla}(t)$ as $\nabla \rightarrow 0$.

$\therefore x(t)\delta(t) = x(0)\delta(t)$.

Similarly by same argument,
$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$