EEEB233 : SIGNALS AND SYSTEMS

SEMESTER I 15/16

TEST 2
90 Minutes (28TH AUGUST 2015)

Name : …………………………………………………
ID No. : …………………………………………………
Section : …………………………………………………

<table>
<thead>
<tr>
<th>Question</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20 MARKS</td>
</tr>
<tr>
<td>2</td>
<td>20 MARKS</td>
</tr>
<tr>
<td>3</td>
<td>20 MARKS</td>
</tr>
<tr>
<td>Total</td>
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</tr>
</tbody>
</table>

Reminder:
1. Do not open the question paper until you are instructed to do so.
2. Answer all questions. Show all your workings for full credit.
3. The use of pencil and liquid paper are not allowed.
4. This question paper consists of 9 pages including this page.

Good Luck!
QUESTION 1a [10 MARKS]

The Fourier series representation of a periodic signal $x(t)$ is given as:

$$x(t) = \frac{1}{4} + \sum_{k=-2}^{2} \frac{1}{2jk} e^{-jk\frac{\pi}{3}t}$$

i) Find the exponential Fourier series coefficients of $x(t)$. [2 marks]

ii) Express $x(t)$ in trigonometric form. [3 marks]
iii) Find the exponential Fourier series coefficients of \( x(-t) \). [2 marks]

iv) Calculate the exponential second harmonic of the output if the signal \( x(t) \) is applied as input to the circuit shown in Figure 1 where \( R = 1 \) ohm \( C = 1 \) F.  [3 marks]

![Figure 1](image)

The frequency response of the given circuit is

\[
H(jk\omega_0) = \frac{1}{\frac{jk\omega_0C}{1+jk\omega_0CR} + R} = \frac{1}{1+jk\omega_0CR}
\]

By substituting \( k = 2, R = 1 \) and \( C = 1, \omega_0 = \frac{\pi}{3} \)

The eigenvalue of the second harmonic is

\[
\frac{1}{1+j2\frac{\pi}{3}} = \frac{1}{1+j2.1}
\]

Let \( b_2 \) be the second harmonic of the output \( y(t) \)

\[
b_2 = a_2 \cdot \frac{1}{1+j2.1} = \frac{1}{4j} \cdot \frac{1}{1+j2.1} = \frac{1}{4j-8.4}
\]
QUESTION 1b [10 MARKS]

Given a linear time invariant system with an impulse response of \( h(t) = 3e^{-2t}u(t) \). The signal \( x(t) \) from Question 1 part (a) is applied as an input to the system.

i) Determine the frequency response of the system. [5 marks]

ii) Hence, find the output \( y(t) \) of the system in exponential Fourier series representation. [5 marks]
QUESTION 2a [13 MARKS]

Given the following periodic discrete-time signal

\[ x[n] = 2 + \sin \left( \frac{n\pi}{2} + \frac{3\pi}{2} \right) + \cos \left( \pi n + \frac{\pi}{4} \right) \]

i. Find the fundamental period of the signal. [2 marks]

ii. Find the exponential Fourier series coefficients, \( a_k \) of the signal. [4 marks]
iii. Plot the magnitude and the phase of the exponential Fourier series coefficients. [5 marks]

iv. Calculate the total average power of the signal. [2 marks]
QUESTION 2b [7 MARKS]

Consider the discrete-time periodic signal $x[n]$ shown in Figure 2.

![Figure 2](image)

i. Find the exponential Fourier series representation of the signal. [6 marks]

ii. Determine the $15^{th}$ harmonic of the signal $x[n]$, $a_{15}$ [1 mark]
QUESTION 3a [8 MARKS]

Given the signal $x(t)$ shown in Figure 3. Calculate its Fourier transform

\[ Y(j\omega) = -2e^{j2\omega} - 2e^{-j2\omega} + 4\text{sinc}2\omega \]  

[2 marks]

\[ Y(j\omega) = -4\cos2\omega + 4\text{sinc}2\omega \]  

[1 marks]

\[ X(j\omega) = \frac{-4\cos2\omega + 4\text{sinc}2\omega}{j\omega} \]  

[1 marks]
QUESTION 3b [12 MARKS]

i) The input and output of an LTI system are related by the difference equation:
\[
\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2 = \frac{dx(t)}{dt}.
\]
Calculate the impulse response \( h(t) \) of the system [4 marks]

\[
H(j\omega) = \frac{j\omega}{(j\omega)^2 + 3(j\omega) + 2} = \frac{j\omega}{(j\omega + 2)(j\omega + 1)} = \frac{2}{(j\omega + 2)} + \frac{-1}{(j\omega + 1)}
\] [2 marks]

The impulse response \( h(t) = 2e^{-2t}u(t) - e^{-t}u(t) \) [2 marks]

ii) If an input signal \( x(t) = 2e^{-t}u(t) \) is applied to the system in part (i), calculate the output signal \( y(t) \) [8 marks]

\[
H(j\omega) = \frac{j\omega}{(j\omega)^2 + 3(j\omega) + 2} = \frac{j\omega}{(j\omega + 2)(j\omega + 1)} = \frac{2}{(j\omega + 2)} + \frac{-1}{(j\omega + 1)}
\] [2 marks]

\[
X(j\omega) = \frac{2}{1 + j\omega}
\] [1 mark]

\[
Y(j\omega) = X(j\omega)H(j\omega) = \frac{2}{1+j\omega} \left( \frac{2}{(j\omega+2)} + \frac{-1}{(j\omega+1)} \right)
\] [1 marks]

\[
Y(j\omega) = \frac{4}{(j\omega + 2)(j\omega + 1)} - \frac{2}{(1+j\omega)^2} = \frac{-4}{2 + j\omega} + \frac{4}{1 + j\omega} - \frac{2}{(1 + j\omega)^2}
\] [2 marks]

\[
y(t) = -4e^{-2t}u(t) + 4e^{-t}u(t) - 2te^{-t}u(t)
\]